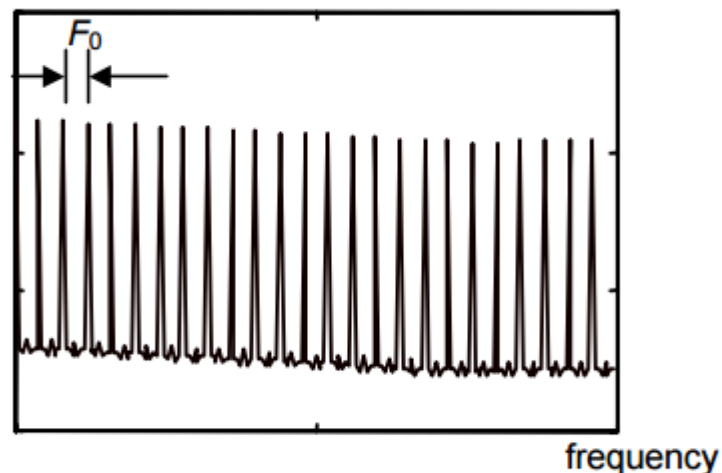
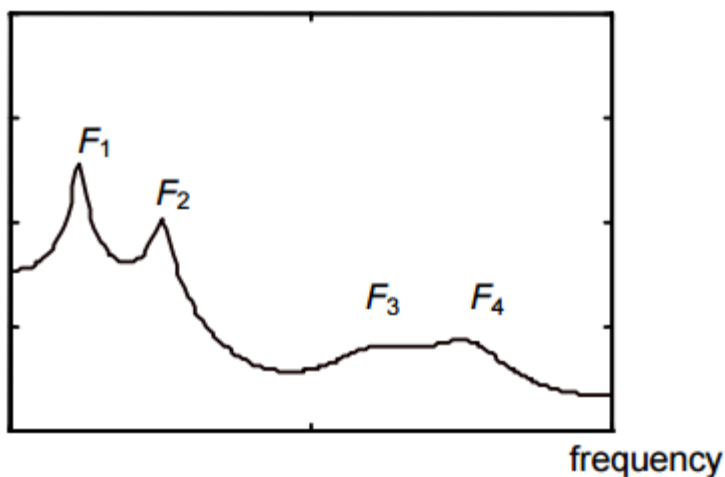


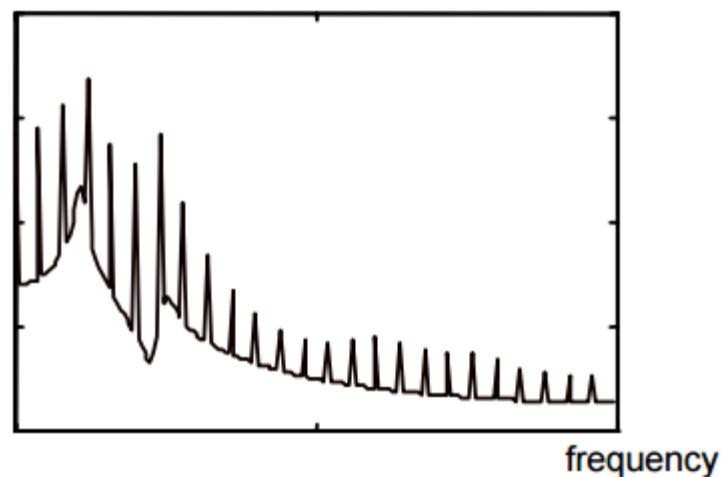
(a) Vocal tract excitation



(b) Spectrum of input (dB)



(c) Vocal tract freq. response (dB)



(d) Spectrum of speech (dB)

$$r[n] = s[n] + \sum_{k=1}^p a_k s[n-k]$$

$$\min_{\{a_k\}} \sum_n r^2[n] = J$$

$$\sum_n \left(s[n] + \sum_{k=1}^p a_k s[n-k] \right)^2$$

$$= \sum_n s^2[n] + 2 \sum_n s[n] \sum_{k=1}^p a_k s[n-k] + \sum_n \left(\sum_{k=1}^p a_k s[n-k] \right) \left(\sum_{l=1}^p a_l s[n-l] \right)$$

$$= \sum_n s^2[n] + 2 \sum_{k=1}^p a_k \sum_n s[n] s[n-k] + \sum_{k=1}^p \sum_{l=1}^p a_k a_l \sum_n s[n-k] s[n-l]$$

$$\frac{\partial J}{\partial a_j} = 2 \sum_n s[n-j] \left(s[n] + \sum_{k=1}^p a_k s[n-k] \right) = 0$$

$$\Rightarrow \left[\sum_{k=1}^p a_k \sum_n s[n-k] s[n-j] = - \sum_n s[n] s[n-j] \right]$$

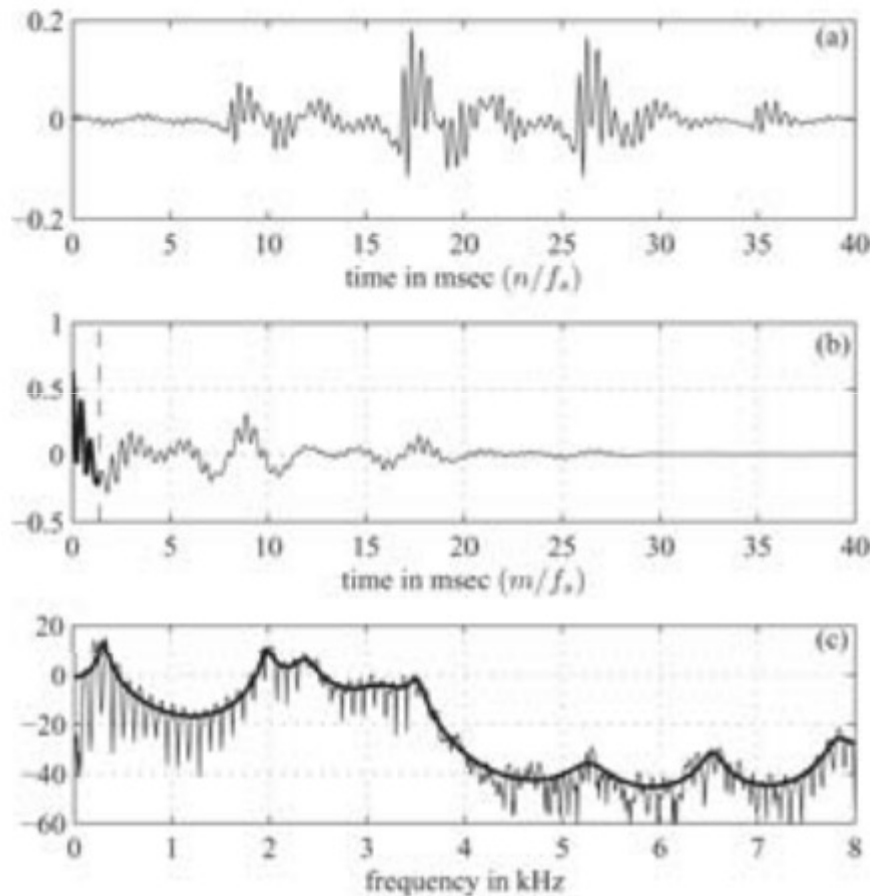
$$J^* = \sum_n s^2[n] + 2 \sum_{k=1}^p a_k \sum_n s[n] s[n-k] - \sum_{l=1}^p a_l \sum_n s[n] s[n-l]$$

$$\Rightarrow J^* = \underbrace{\sum_n s^2[n]}_{R_{xx}(0)} + \sum_{k=1}^p a_k \underbrace{\sum_n s[n] s[n-k]}_{R_{xx}(k)}$$

In matrix form the set of linear equation can be expressed as:

$$\begin{bmatrix} R(0) & R(1) & R(2) & \dots & R(p-1) \\ R(1) & R(0) & R(1) & \dots & R(p-2) \\ R(2) & R(1) & R(0) & \dots & R(p-3) \\ \dots & \dots & \dots & \dots & \dots \\ R(p-1) & R(p-2) & R(p-3) & \dots & R(0) \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \dots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \\ R(3) \\ \dots \\ R(p) \end{bmatrix} \quad (-1)$$

LP Short-Time Spectrum Analysis

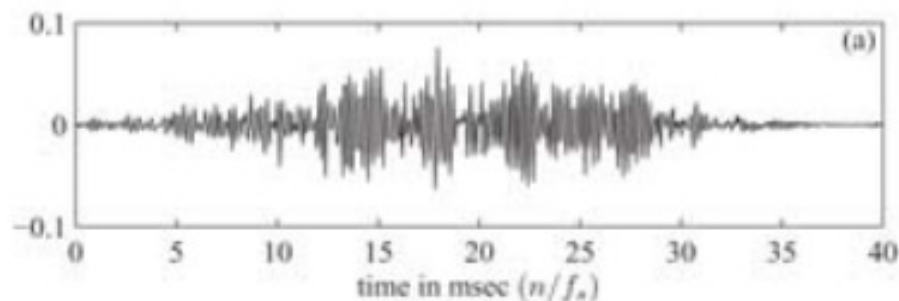


(a) Voiced speech segment obtained using a Hamming window

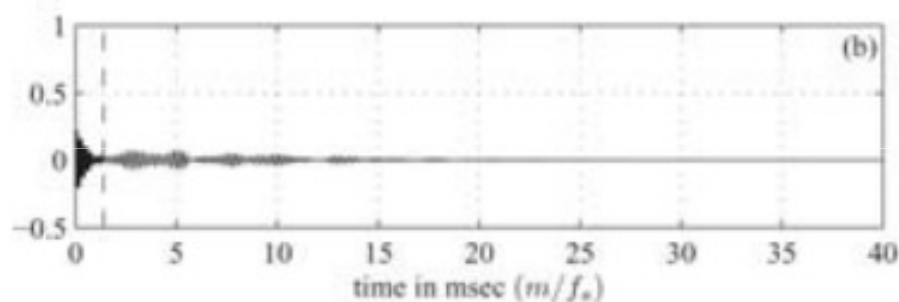
(b) Corresponding short-time autocorrelation function used in LP analysis (heavy line shows values used in LP analysis)

(c) Corresponding short-time log magnitude Fourier transform and short-time log magnitude LPC spectrum ($F_s=16$ kHz)

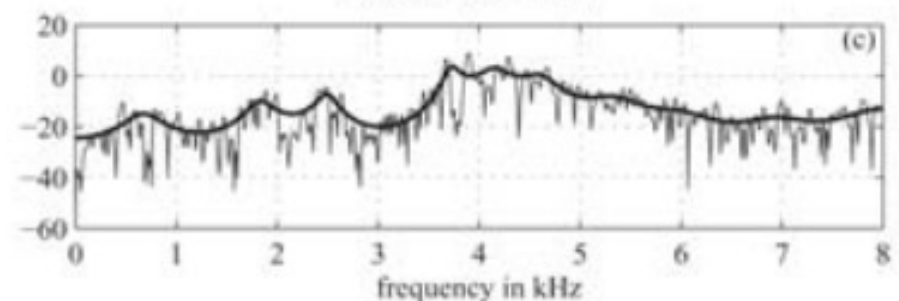
LP Short-Time Spectrum Analysis



(a) Unvoiced speech segment obtained using a Hamming window

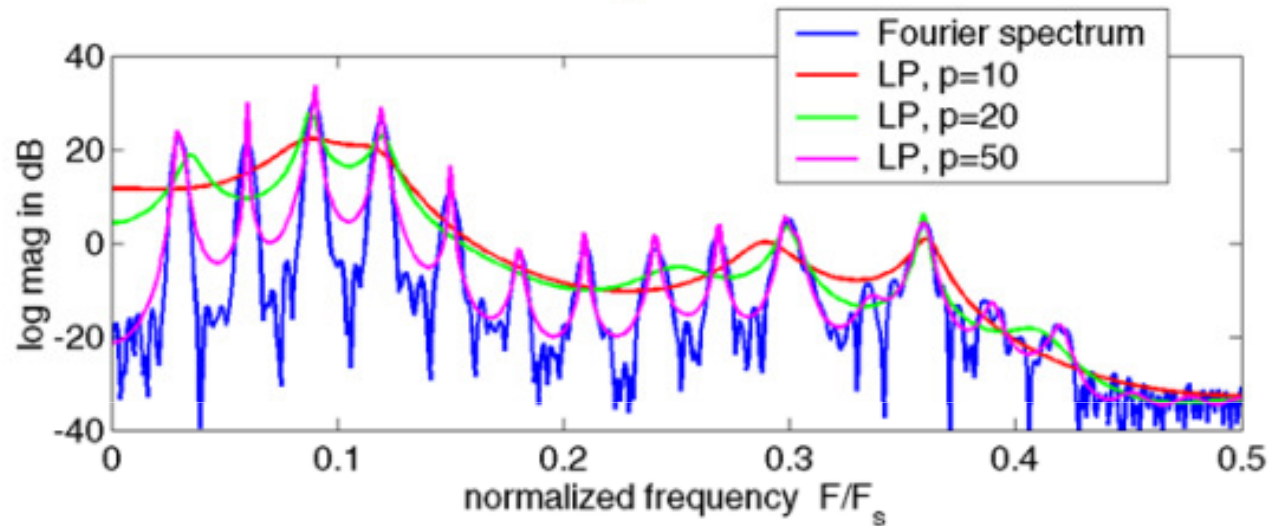


(b) Corresponding short-time autocorrelation function used in LP analysis (heavy line shows values used in LP analysis)



(c) Corresponding short-time log magnitude Fourier transform and short-time log magnitude LPC spectrum ($F_s=16$ kHz)

LPC Spectrum

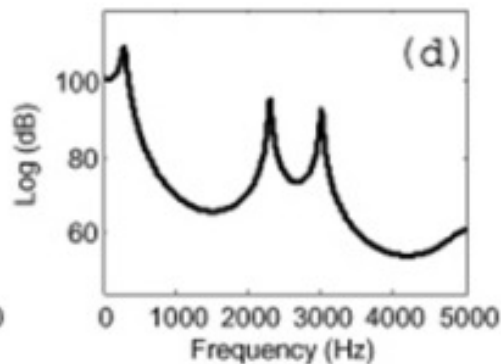
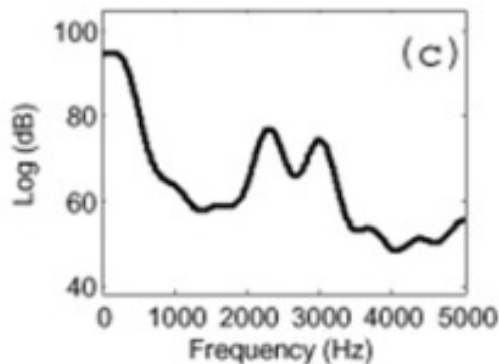
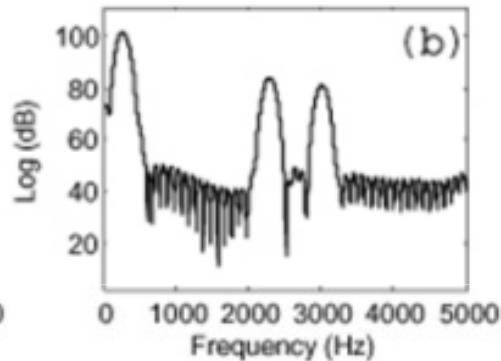
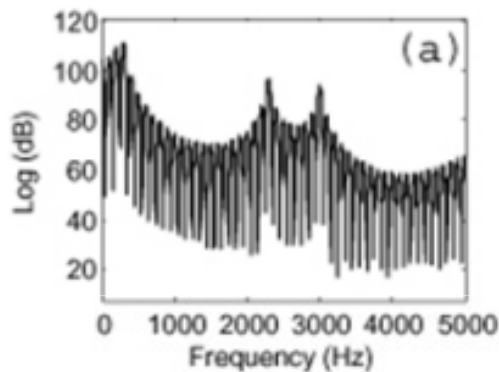


$$\tilde{H}(e^{j\omega}) = \frac{G}{1 - \sum_{k=1}^p \alpha_k e^{-j\omega k}}$$

```
x = s .* hamming(301);  
X = fft( x , 1000 )  
[ A , G , r ] = autolpc( x , 10 )  
H = G ./ fft(A,1000);
```

LP Analysis is seen to be a method of short-time spectrum estimation with removal of excitation fine structure (a form of wideband spectrum analysis)

Comparison to Other Spectrum Analysis Methods



Spectra of synthetic vowel /IY/

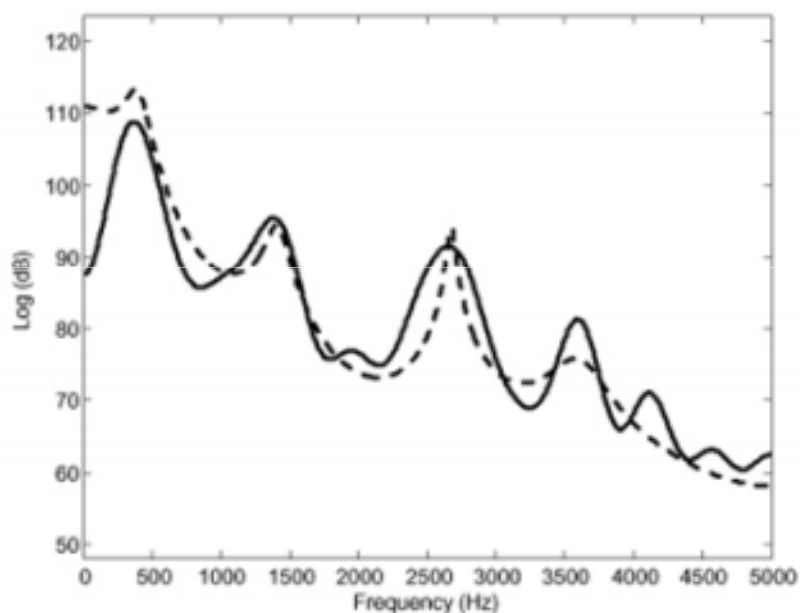
(a) Narrowband spectrum using 40 msec window

(b) Wideband spectrum using a 10 msec window

(c) Cepstrally smoothed spectrum

(d) LPC spectrum from a 40 msec section using a $p=12$ order LPC analysis

Comparison to Other Spectrum Analysis Methods



Natural speech spectral estimates using cepstral smoothing (solid line) and linear prediction analysis (dashed line).

Note the fewer (spurious) peaks in the LP analysis spectrum since LP used $p=12$ which restricted the spectral match to a maximum of 6 resonance peaks.

Note the narrow bandwidths of the LP resonances versus the cepstrally smoothed resonances.

