$$u[n] \longrightarrow \boxed{\frac{B(z)}{A(z)}} \longrightarrow x[n]$$

$$x[n] = -\sum_{k=1}^{p} a_k\, x[n-k] + \sum_{k=0}^{q} b_k\, u[n-k]$$

$$B(z) = \sum_{k=0}^{q} b_k z^{-k}$$

$$A(z) = \sum_{k=0}^{p} a_k z^{-k}$$

$$H(z) = \frac{B(z)}{A(z)} = \sum_{n=0}^{\infty} h(n) z^{-k}$$

causal assumed.

$$x[n] = \sum_{k=0}^{\infty} h_k\, u[n-k]$$

Let $u[n]$ is white noise

$$R_{uu}[l] = \delta[l]\,\sigma^2$$

$$R_{xx}[l] = \sum_n x[n]\, x[n-l]$$

$$= -\sum_{k=1}^{p} a_k R_{xx}[l-k] + \sum_{k=0}^{q} b_k R_{ux}[l-k]$$

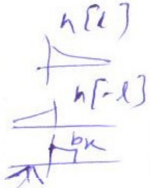
But

$$R_{ux}[l] = \sum_n u[n]\, x[n-l] = \sum_{k=0}^{\infty} h_k \sum_n u[n]\, u[n-l-k]$$

$$= \sum_{k=0}^{\infty} h_k\, \delta[k+l]\cdot\sigma^2 = h[-l]\,\sigma^2 \quad \forall l.$$

So

$$\boxed{R_{xx}[l] = -\sum_{k=1}^{p} a_k R_{xx}[l-k] + \sigma^2 \sum_{k=l}^{q} b_k\, h[k-l]} \quad \forall l$$




for $l > q$ $(q+1 \cdots q+p)$

$$R_{xx}[l] = -\sum_{k=1}^{p} a_k R_{xx}[l-k]$$

~~modified~~ Extended YWE.

from where $\{a_k\}$ can be solved but need to know '$q$'.

AR E – LD

CNE – Cholesky

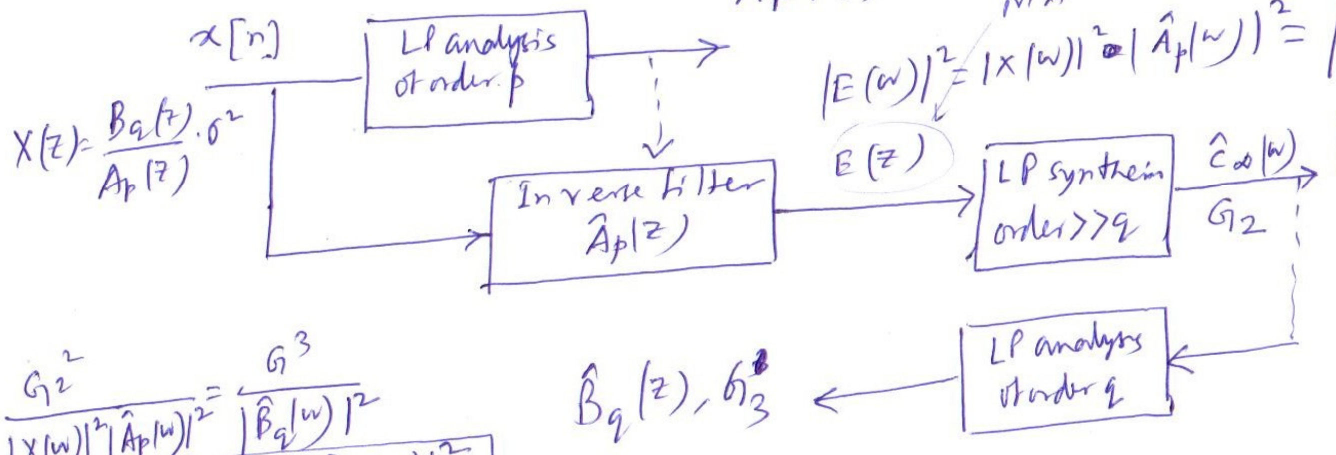
EYWE – Toeplitz but not symm (take inv.)

For solv $b_k$

$$R_{xx}[l] = -\sum_{k=1}^{p} a_k R_{xx}[l-k] + \underbrace{\sigma^2 \sum_{k=l}^{q} b_k\, h[k-l]}_{\text{non linear}} \quad l = 1, \cdots q$$

So kolmogrov relⁿ can be used.

Sequential soln to ARMA




MA

$$|E(\omega)|^2 = |X(\omega)|^2 |\hat{A}_p(\omega)|^2 = \left|\frac{B_q(\omega)}{A_p(\omega)}\right|^2 |\hat{A}_p(\omega)|^2 \approx |\hat{B}_q(\omega)|^2$$

$$|\hat{C}_\infty(\omega)|^2 = \frac{G_2^2}{|\hat{B}_q(\omega)|^2} \simeq \frac{G_3^2}{|\hat{B}_q(\omega)|^2}$$

$$\frac{G_2^2}{|X(\omega)|^2 |\hat{A}_p(\omega)|^2} = \frac{G^3}{|\hat{B}_q(\omega)|^2}$$

$$\boxed{|X(\omega)|^2 = G^2 \frac{|\hat{B}_q(\omega)|^2}{|\hat{A}_p(\omega)|^2}}$$