E9:261

24-02-2016

Recap ...

- Modeling of Speech
  - Single Gaussian Model
    - Maximum likelihood estimation (MLE) of parameters [Sample mean and sample covariance]
    - Pros - Simplicity. Cons - Not capable of modeling speech like signals.
  - Gaussian Mixture Models
    - MLE is not straight forward

$$p(\mathbf{x}|\theta) = \sum_{i=1}^{M} \alpha_i p_i(\mathbf{x}|\theta_i)$$

    - Expectation Maximization (EM) algorithm.
      - Assuming a hidden variable (identity of the mixture component).

Matrix Differentiation Rules

The trace of a square matrix $\text{tr}(A)$ is equal to the sum of A's diagonal elements. The trace of a scalar equals that scalar. Also, $\text{tr}(A+B) = \text{tr}(A) + \text{tr}(B)$, and $\text{tr}(AB) = \text{tr}(BA)$ which implies that $\sum_i x_i^T A x_i = \text{tr}(AB)$ where $B = \sum_i x_i x_i^T$. Also note that $|A|$ indicates the determinant of a matrix, and that $|A^{-1}| = 1/|A|$.

We'll need to take derivatives of a function of a matrix $f(A)$ with respect to elements of that matrix. Therefore, we define $\frac{\partial f(A)}{\partial A}$ to be the matrix with i, j^{th} entry $[\frac{\partial f(A)}{\partial a_{i,j}}]$ where $a_{i,j}$ is the i, j^{th} entry of A. The definition also applies taking derivatives with respect to a vector. First, $\frac{\partial x^T A x}{\partial x} = (A + A^T)x$. Second, it can be shown that when A is a symmetric matrix:

$$\frac{\partial |A|}{\partial a_{i,j}} = \begin{cases} \mathcal{A}_{i,j} & \text{if } i = j \\ 2\mathcal{A}_{i,j} & \text{if } i \neq j \end{cases}$$

where $\mathcal{A}_{i,j}$ is the i, j^{th} cofactor of A. Given the above, we see that:

$$\frac{\partial \log |A|}{\partial A} = \left\{ \begin{array}{ll} \mathcal{A}_{i,j}/|A| & \text{if } i = j \\ 2\mathcal{A}_{i,j}/|A| & \text{if } i \neq j \end{array} \right\} = 2A^{-1} - \text{diag}(A^{-1})$$

by the definition of the inverse of a matrix. Finally, it can be shown that:

$$\frac{\partial \text{tr}(AB)}{\partial A} = B + B^T - \text{Diag}(B).$$

"A gentle tutorial on EM Algorithm and its application to parameter estimation of GMM and HMM", Jeff Bilmes

EM Algorithm

$$Q(\Theta, \Theta^{(i-1)}) = E\left[\log p(\mathcal{X}, \mathcal{Y}|\Theta)|\mathcal{X}, \Theta^{(i-1)}\right]$$

$$E\left[\log p(\mathcal{X}, \mathcal{Y}|\Theta)|\mathcal{X}, \Theta^{(i-1)}\right] = \int_{\mathbf{y}\in\Upsilon} \log p(\mathcal{X}, \mathbf{y}|\Theta) f(\mathbf{y}|\mathcal{X}, \Theta^{(i-1)}) d\mathbf{y}.$$

$$\Theta^{(i)} = \underset{\Theta}{\operatorname{argmax}}\ Q(\Theta, \Theta^{(i-1)}).$$

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GMM Re-estimation Formulas

$$\alpha_\ell^{new} = \frac{1}{N}\sum_{i=1}^{N} p(\ell|x_i, \Theta^g)$$

$$\mu_\ell^{new} = \frac{\sum_{i=1}^{N} x_i p(\ell|x_i, \Theta^g)}{\sum_{i=1}^{N} p(\ell|x_i, \Theta^g)}$$

$$\Sigma_\ell^{new} = \frac{\sum_{i=1}^{N} p(\ell|x_i, \Theta^g)(x_i - \mu_\ell^{new})(x_i - \mu_\ell^{new})^T}{\sum_{i=1}^{N} p(\ell|x_i, \Theta^g)}$$

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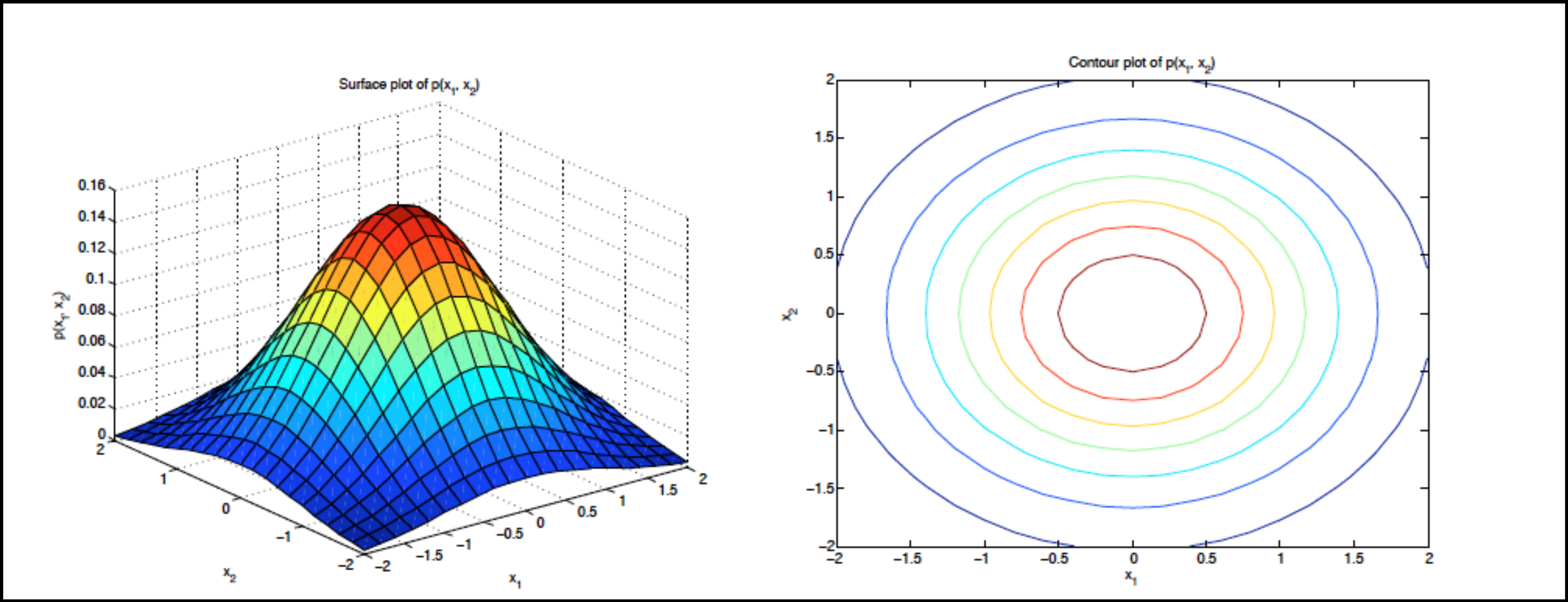
Some Insights - Gaussian Distributions



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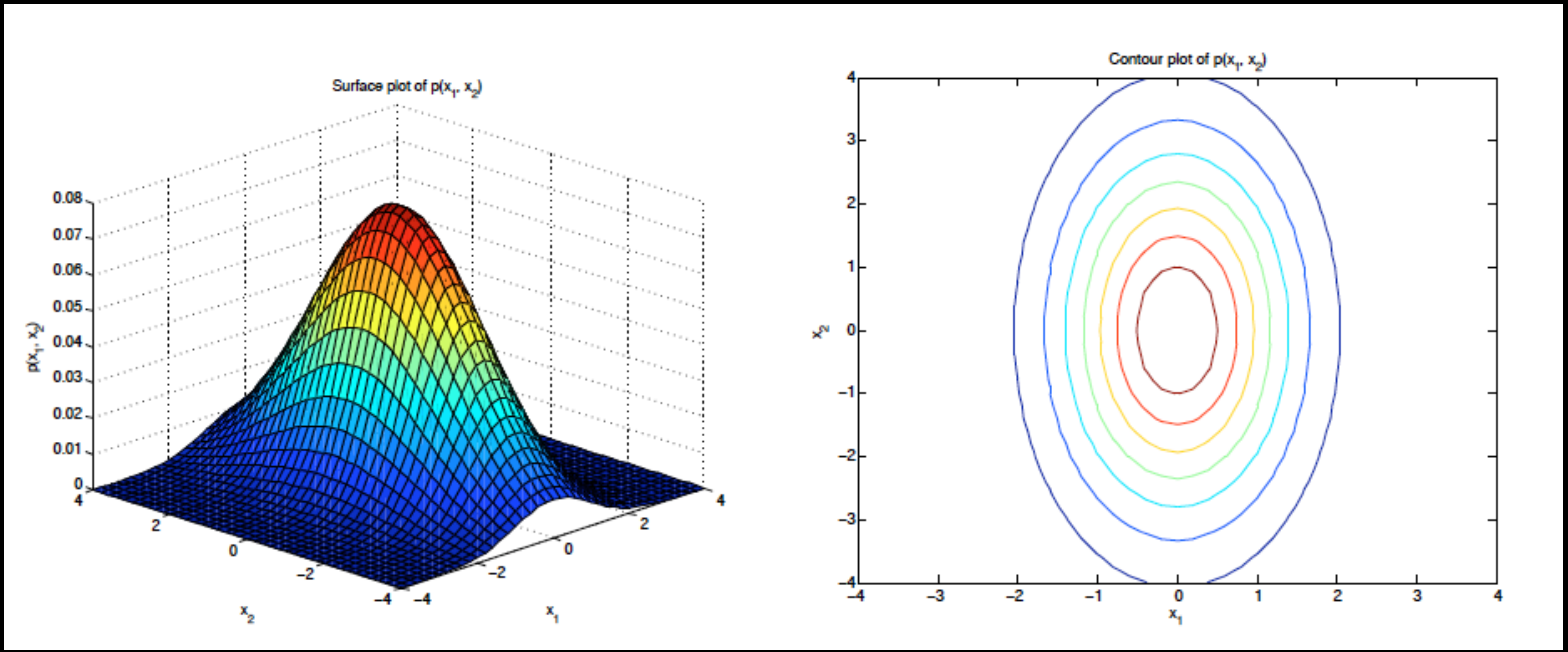

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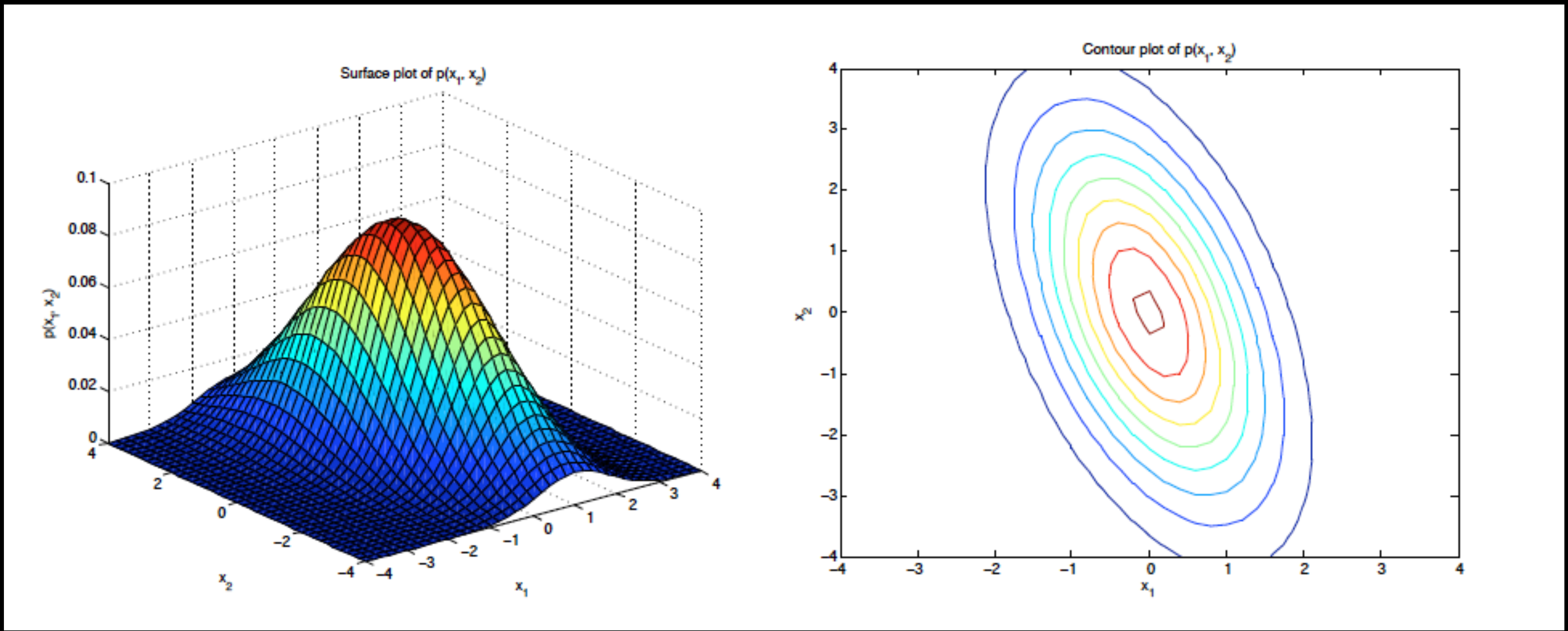

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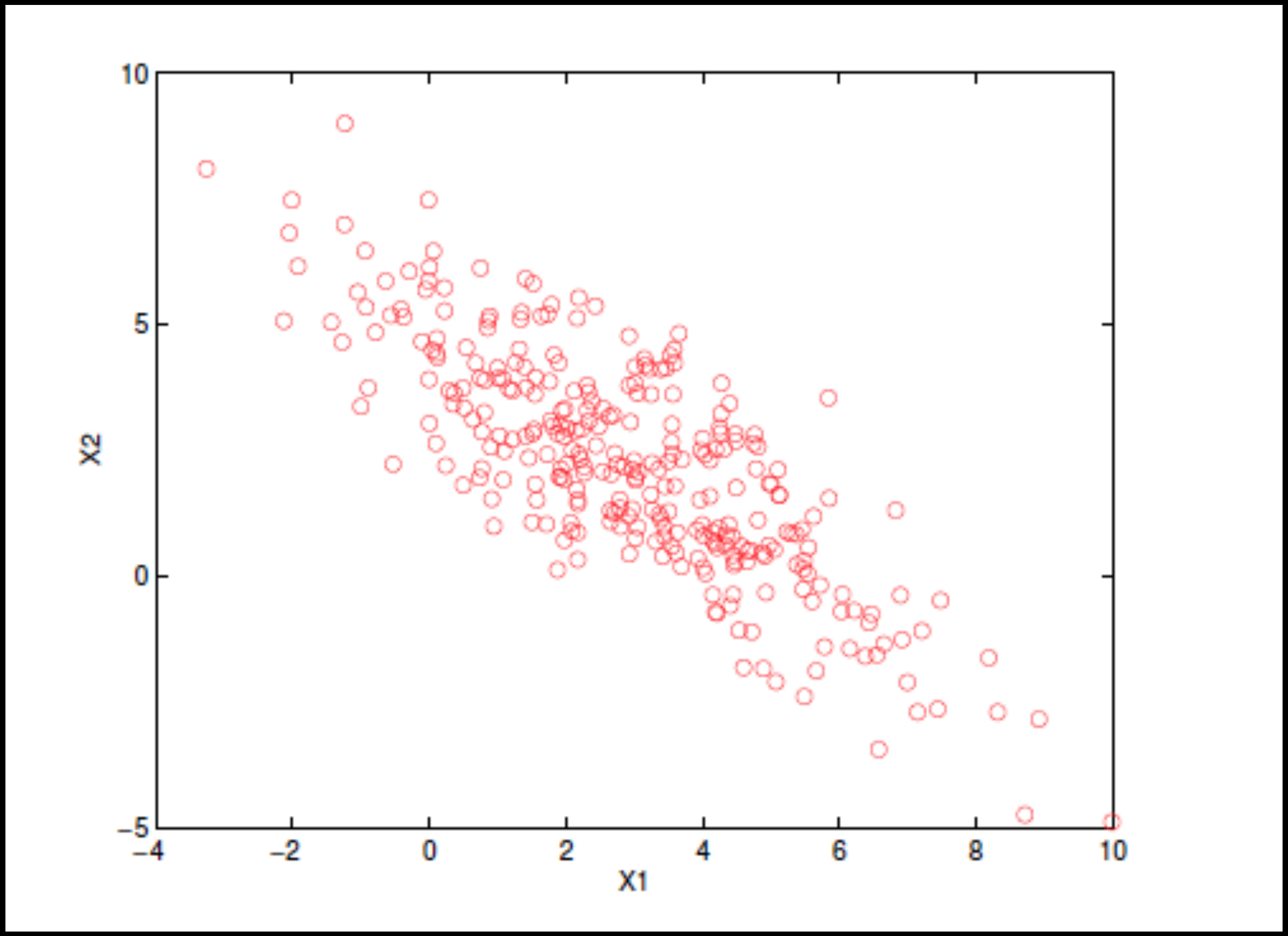

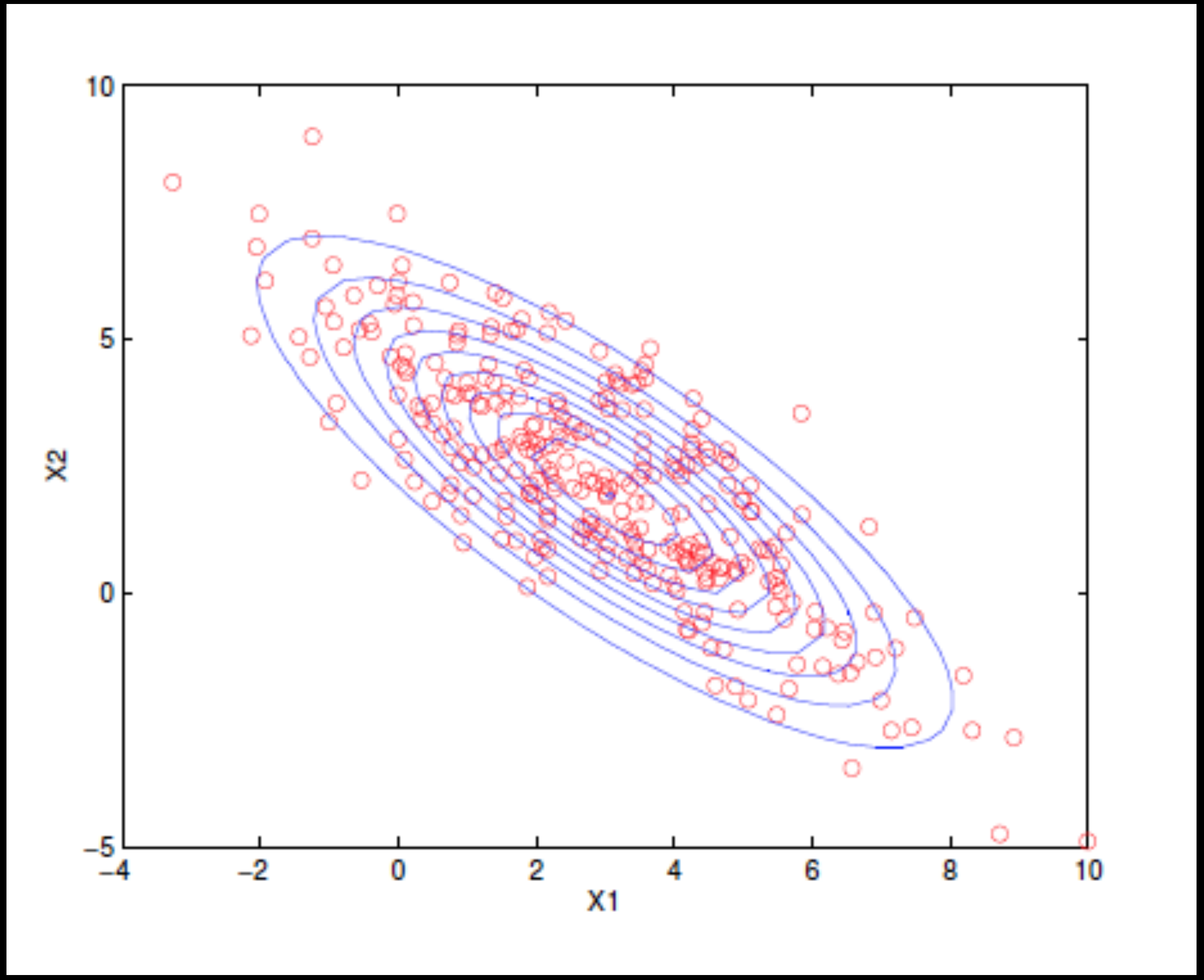

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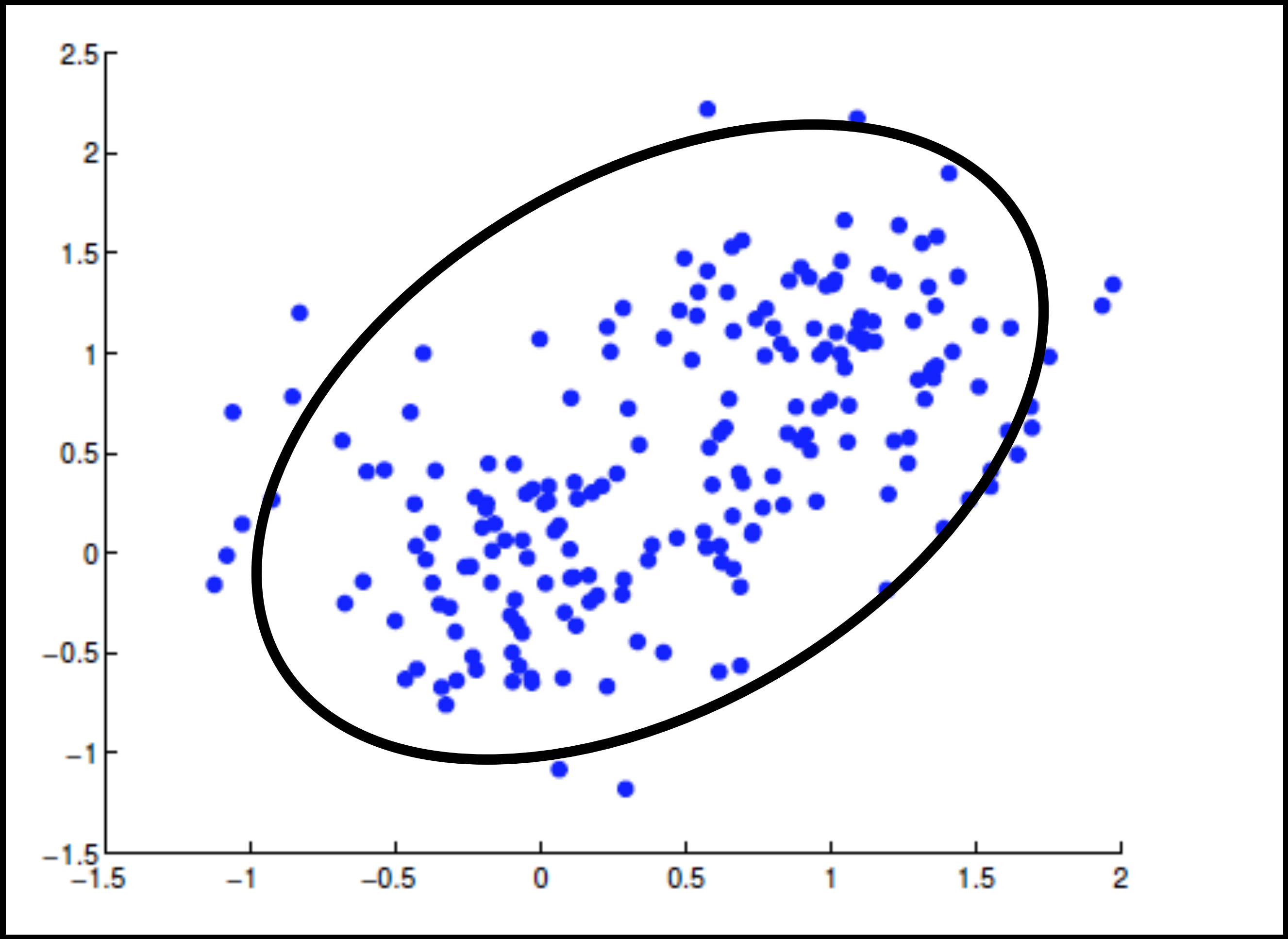

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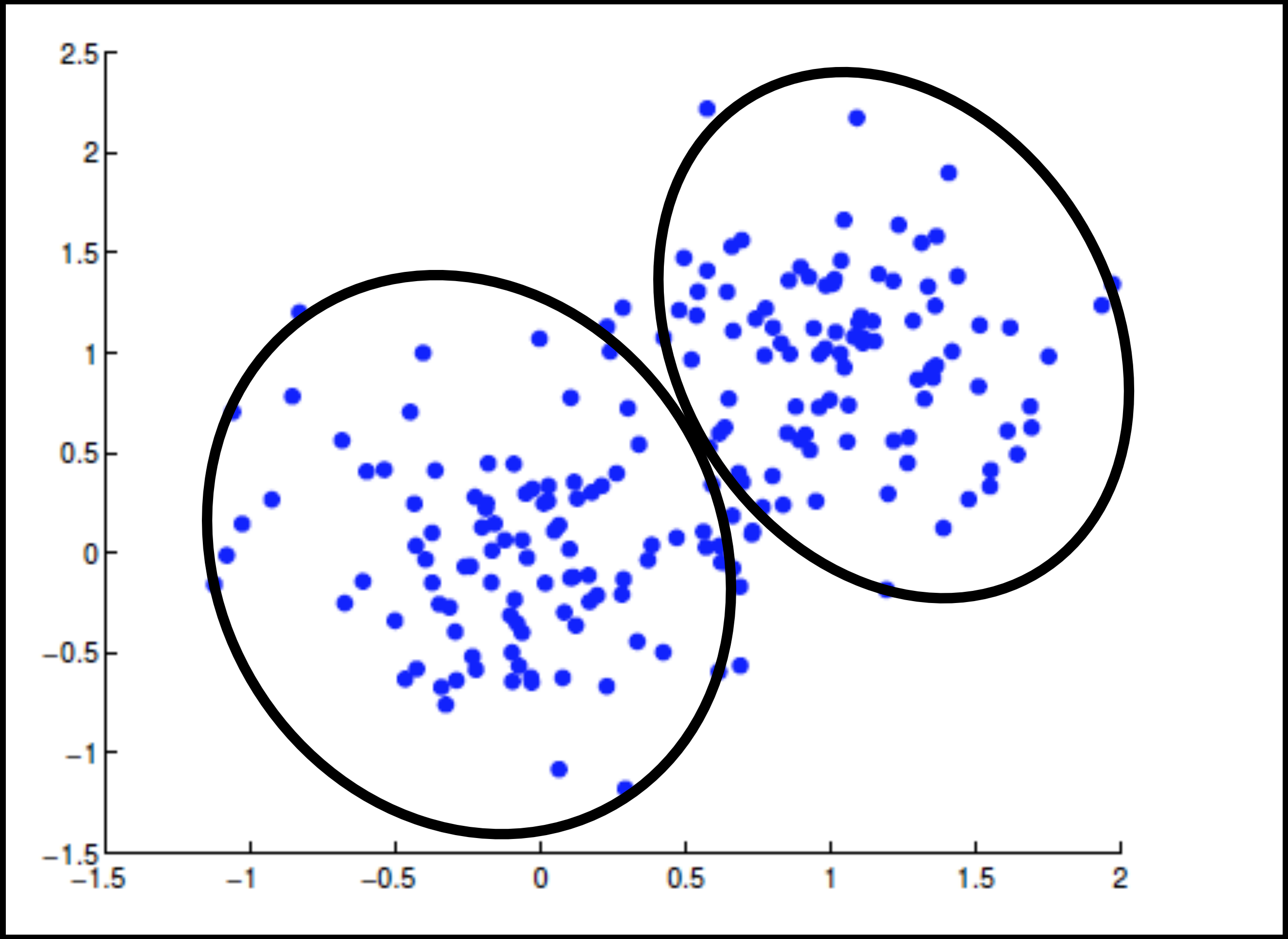

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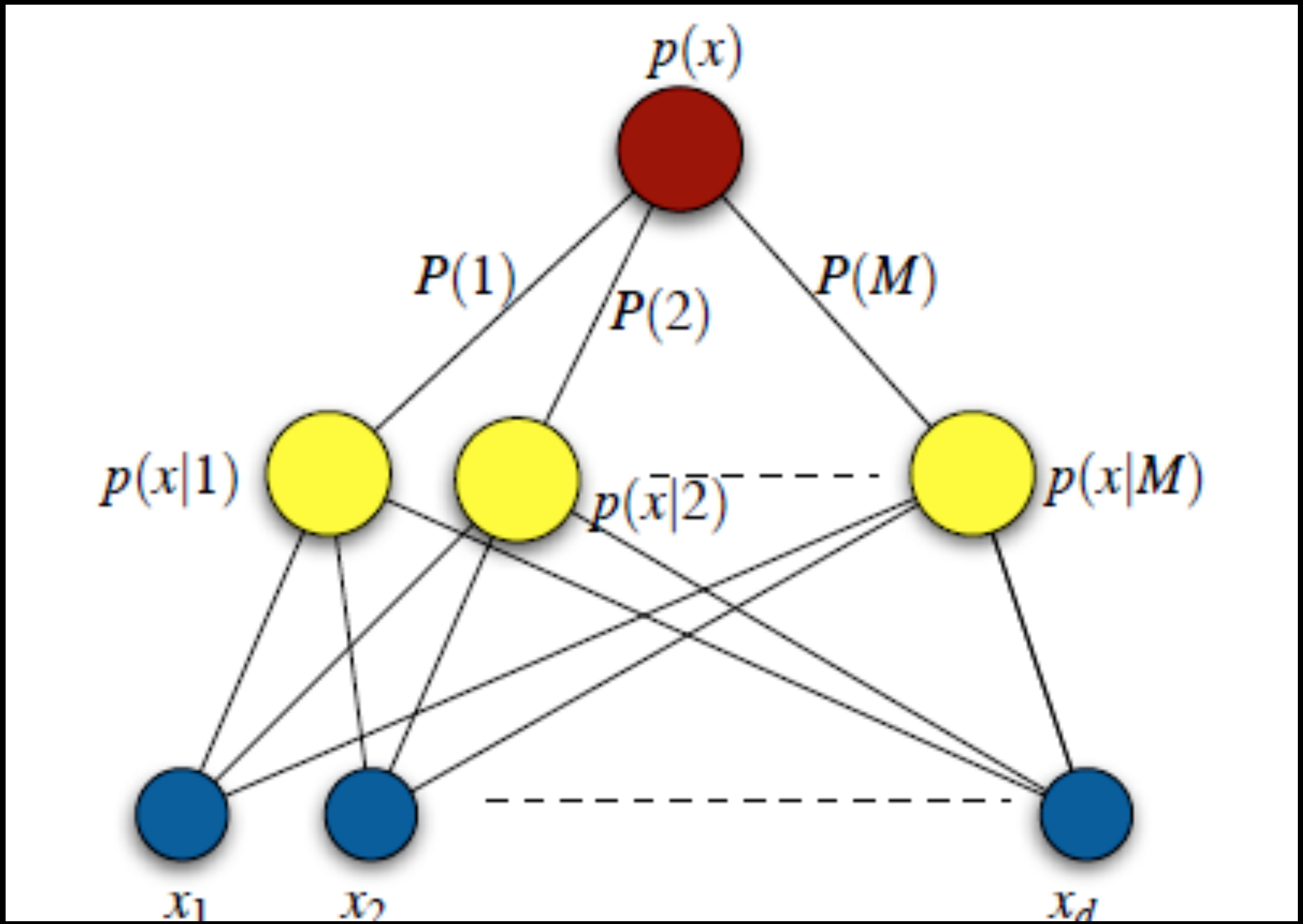
Gaussian Mixture Models



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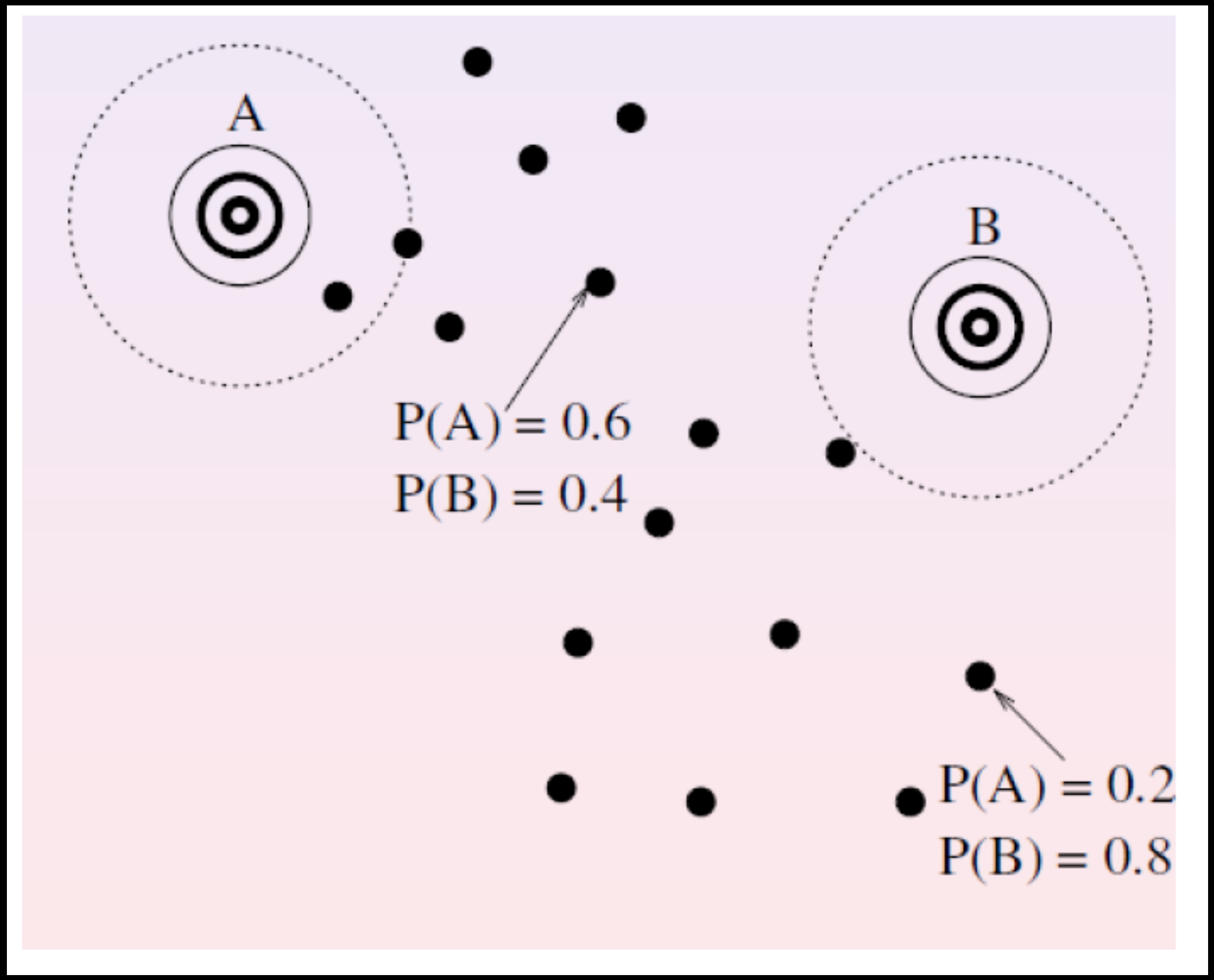
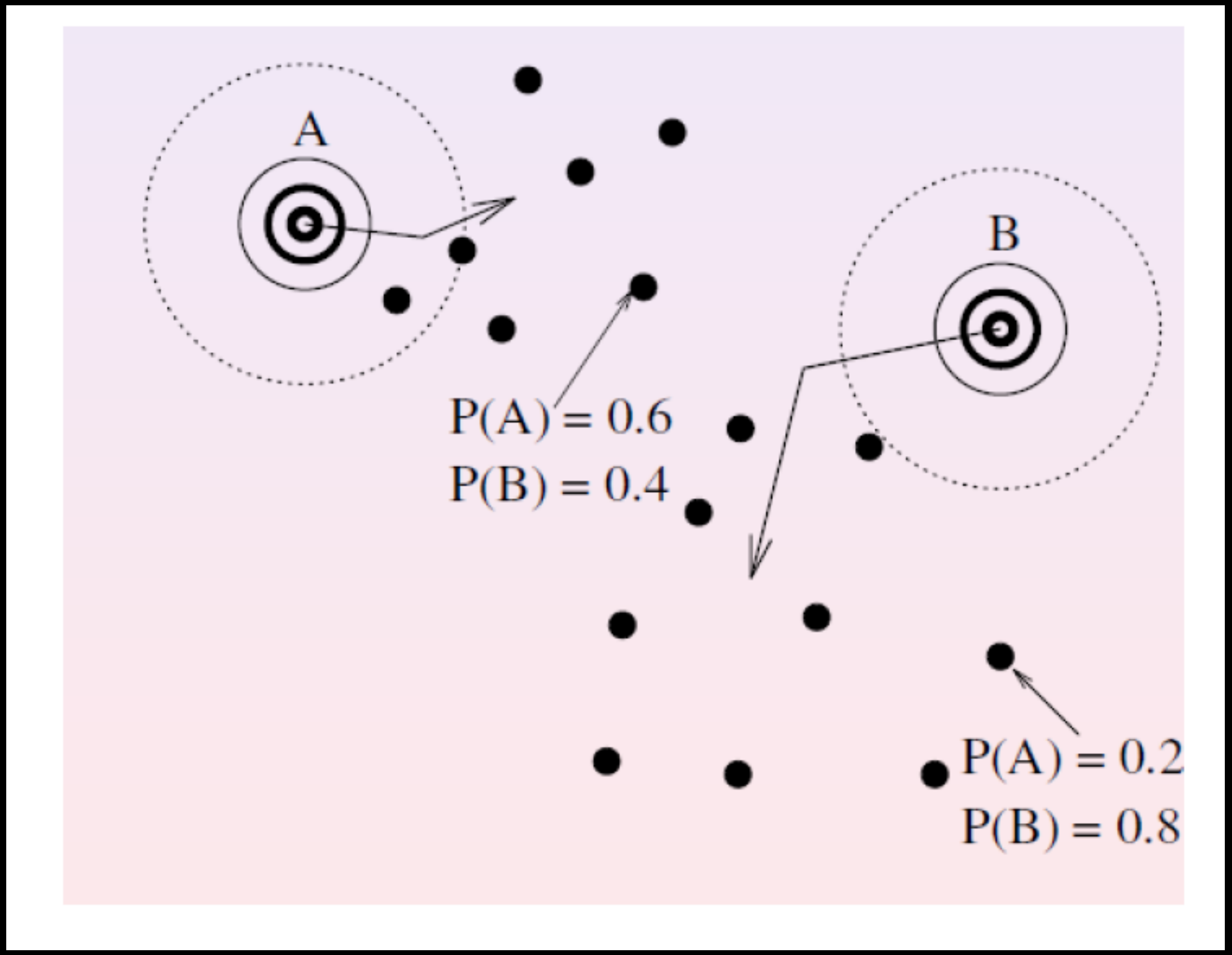
GMM EM Algorithm

E-step



M-step



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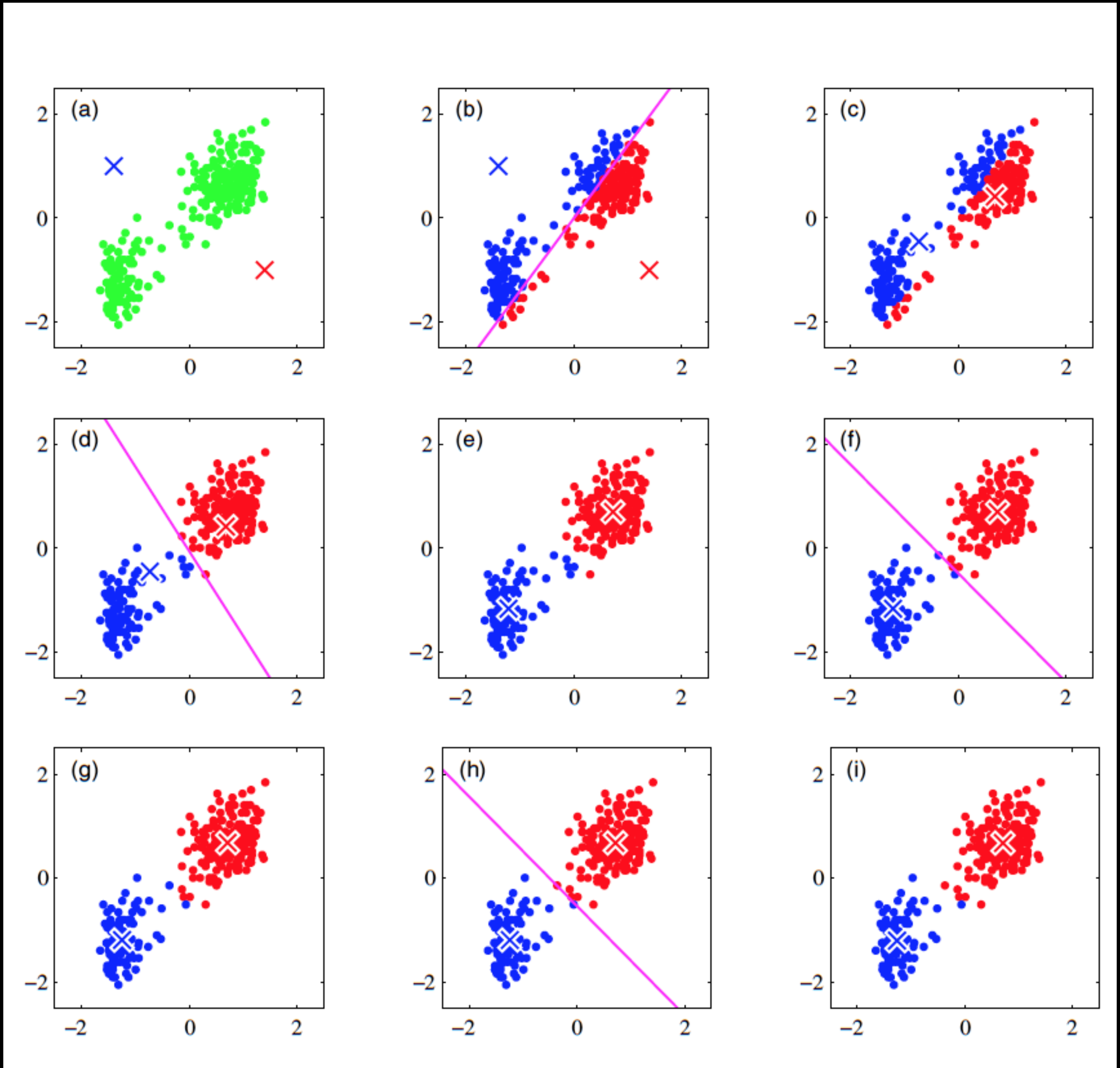
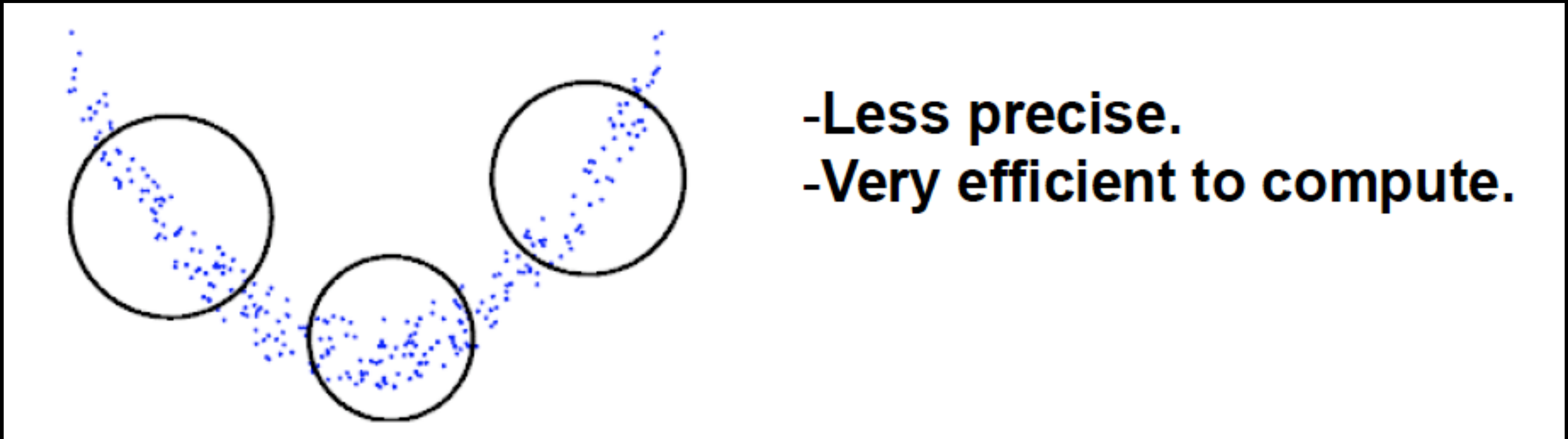
K-means Algorithm



Other Considerations

- Initialization
  - Random Initialization, K-means algorithm, ….
  - Number of Gaussians (Based on the data availability)
  - Type of covariance structure
    - Spherical covariance



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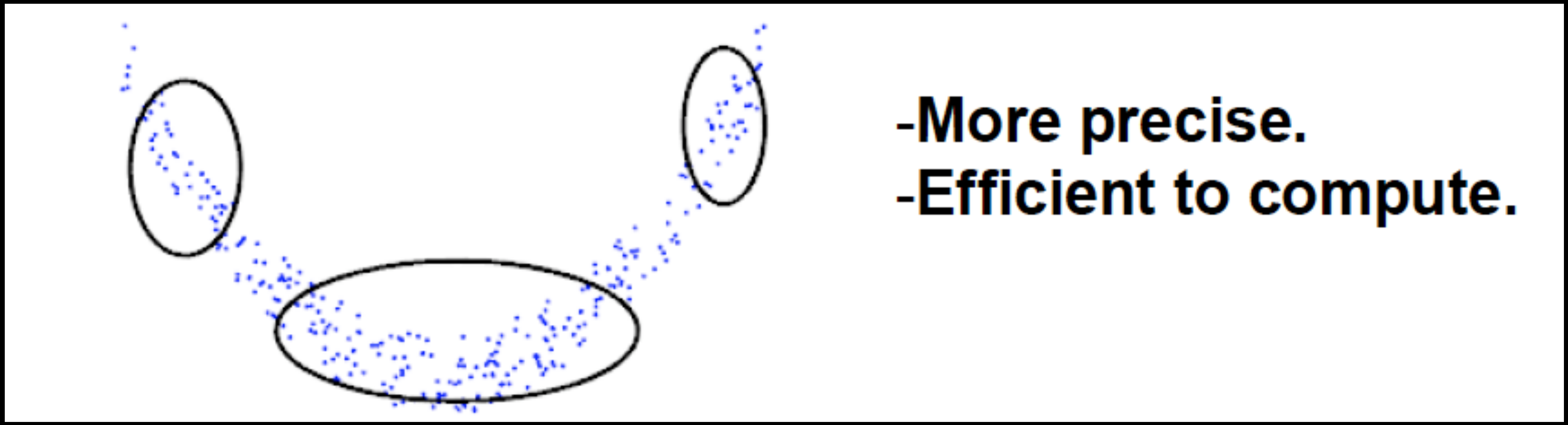
Other Considerations

- Initialization
  - Random Initialization, K-means algorithm, ….
  - Number of Gaussians (Based on the data availability)
  - Type of covariance structure
    - Diagonal covariance



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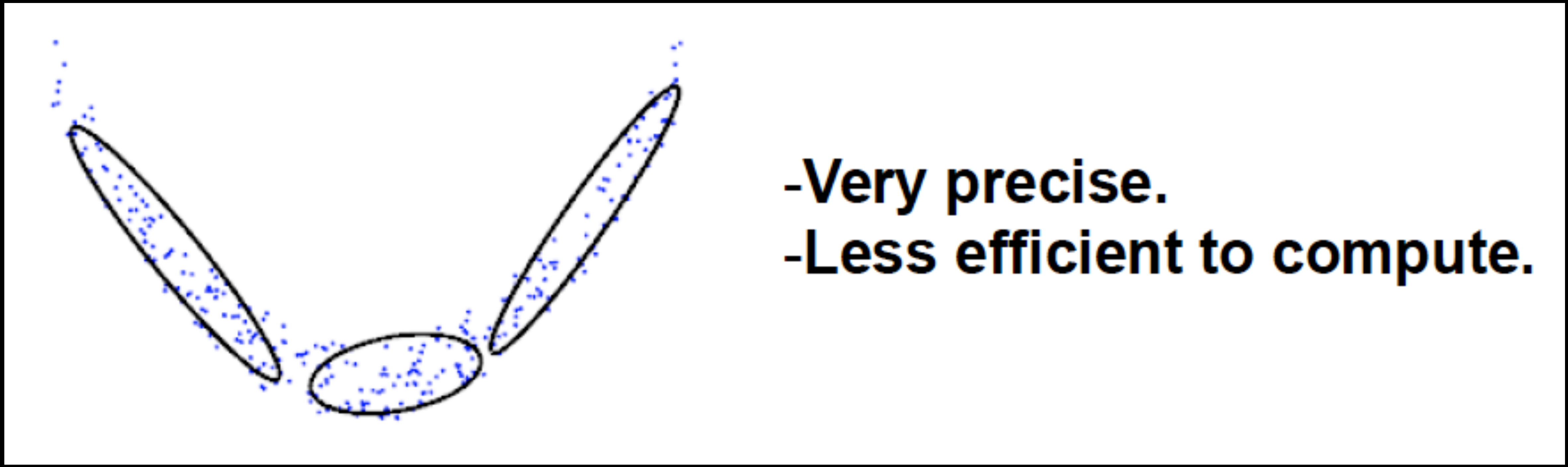
Other Considerations

- Initialization
  - Random Initialization, K-means algorithm, ….
  - Number of Gaussians (Based on the data availability)
  - Type of covariance structure
    - Full covariance



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