# E9 261

09-03-2016

# Recap ...

#### Solution to HMM-GMM re-estimation

$$\pi_i = \frac{\sum_{e=1}^E \gamma_i^e(1)}{E}$$

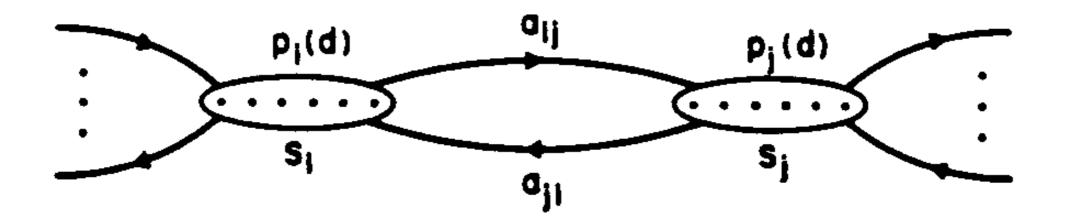
$$c_{i\ell} = \frac{\sum_{e=1}^{E} \sum_{t=1}^{T_e} \gamma_{i\ell}^e(t)}{\sum_{e=1}^{E} \sum_{t=1}^{T_e} \gamma_{i}^e(t)}$$

$$\mu_{i\ell} = \frac{\sum_{e=1}^{E} \sum_{t=1}^{T_e} \gamma_{i\ell}^e(t) o_t^e}{\sum_{e=1}^{E} \sum_{t=1}^{T_e} \gamma_{i\ell}^e(t)}$$

Left to right versus Ergodic HMMs

## Other Considerations

- Implementation issues
  - Scaling
- Explicit duration modeling in HMM



#### **Forward Recursion in Duration Model HMM**

$$\alpha_{t}(i) = \sum_{q} \sum_{d} \pi_{q_{1}} \cdot p_{q_{1}}(d_{1}) \cdot P(\mathbf{o}_{1} \mathbf{o}_{2} \dots \mathbf{o}_{d_{1}} | q_{1})$$

$$\cdot a_{q_{1}q_{2}} p_{q_{2}}(d_{2}) P(\mathbf{o}_{d_{1}+1} \dots \mathbf{o}_{d_{1}+d_{2}} | q_{2}) \dots$$

$$\cdot a_{q_{r-1}q_{r}} p_{q_{r}}(d_{r}) P(\mathbf{o}_{d_{1}+d_{2}+\dots+d_{r-1}+1} \dots \mathbf{o}_{t} | q_{r})$$
(6.67)

where the sum is over all states q and all possible state durations d. By induction we can write  $\alpha_l(j)$  as

$$\alpha_t(j) = \sum_{i=1}^{N} \sum_{d=1}^{D} \alpha_{t-d}(i) a_{ij} p_j(d) \prod_{s=t-d+1}^{t} b_j(\mathbf{o}_s)$$
 (6.68)

where D is the maximum duration within any state. To initialize the computation of  $\alpha_l(j)$  we use

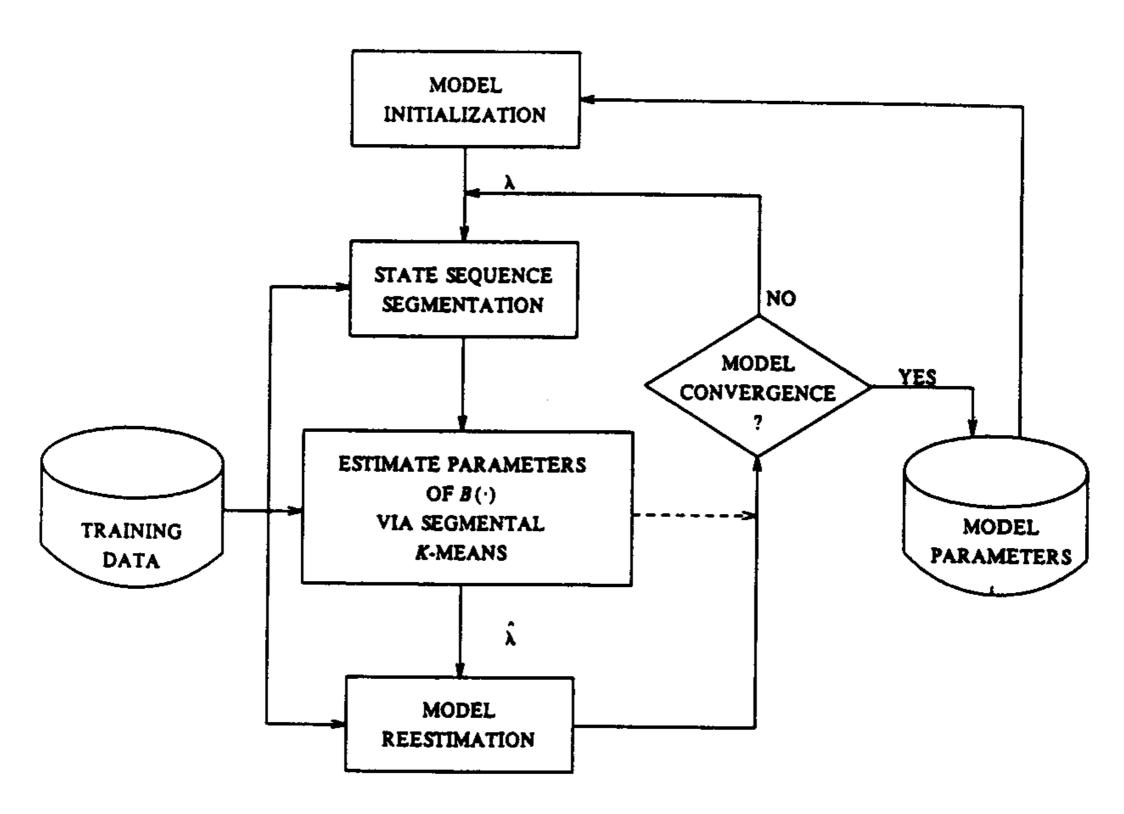
$$\alpha_1(i) = \pi_i p_i(1) \cdot b_i(\mathbf{o}_1) \tag{6.69a}$$

$$P(\mathbf{O}|\lambda) = \sum_{i=1}^{N} \alpha_{T}(i)$$

## Other Considerations

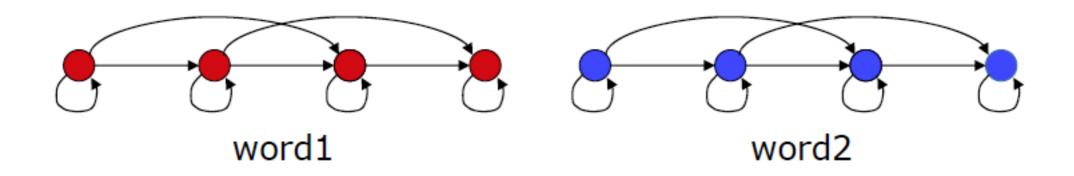
- Implementation issues
  - Scaling
- Explicit duration modeling in HMM
- Comparison of HMMs
- ML versus Bayesian Estimation
- Multiple observation sequence
- Initialization Flat start

## Segmental k-means Algorithm

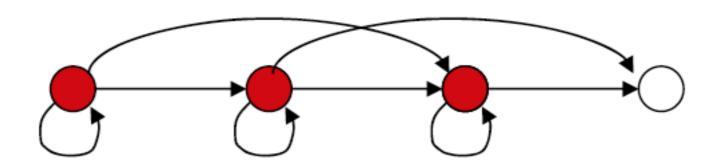


## Dealing with Continuous Speech

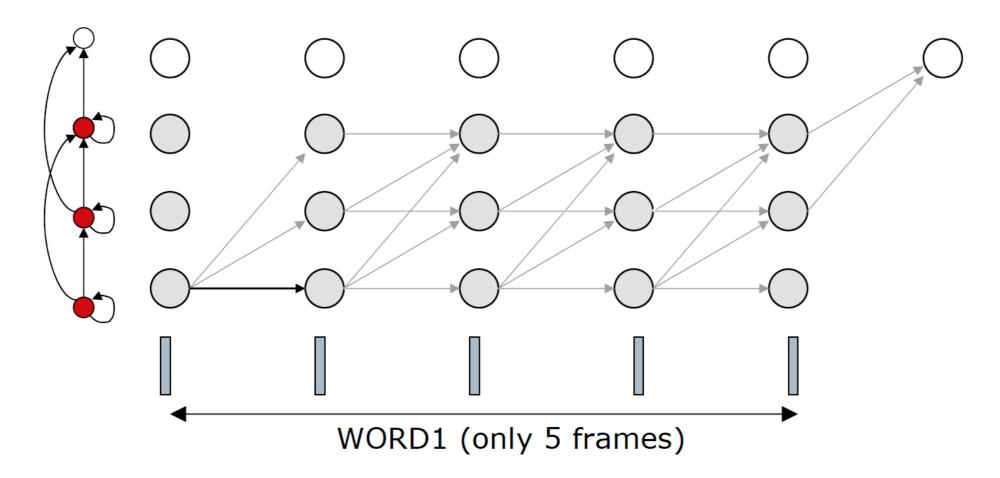
- Isolated Word Recognition is limited
  - Need to deal with string of words
- Word sequences modeled with HMMs which are composed of word HMMs
- Given two words which are both Bakis topology



## Introducing Non-emitting states

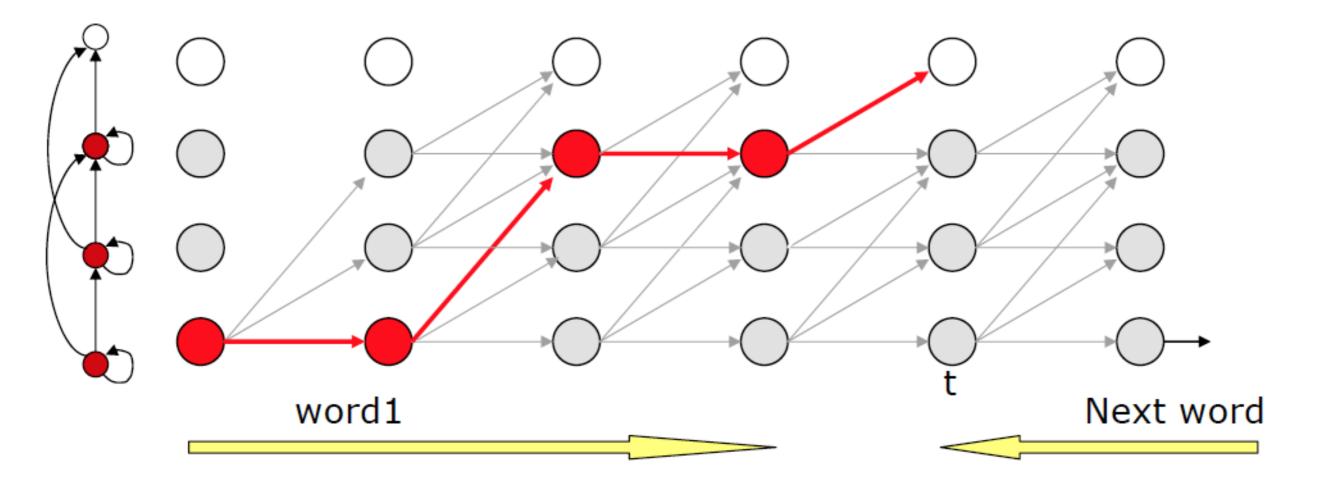


Non-emitting or null states where no observation is emitted



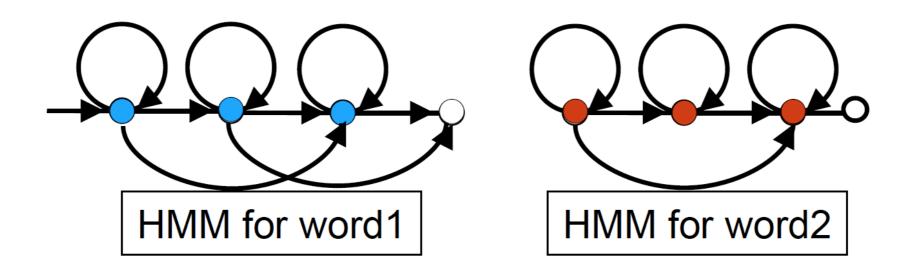
## Introducing Non-emitting states

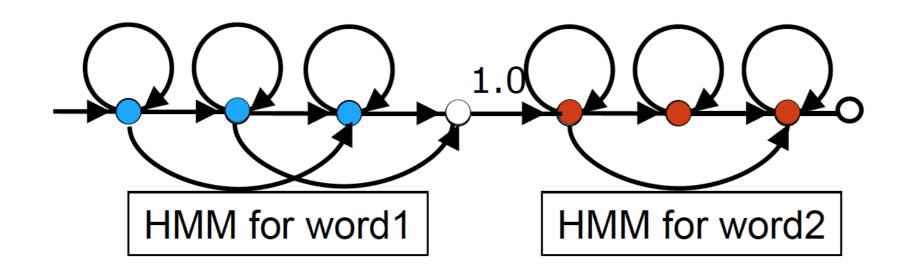
Slight modifications to forward and backward recursion



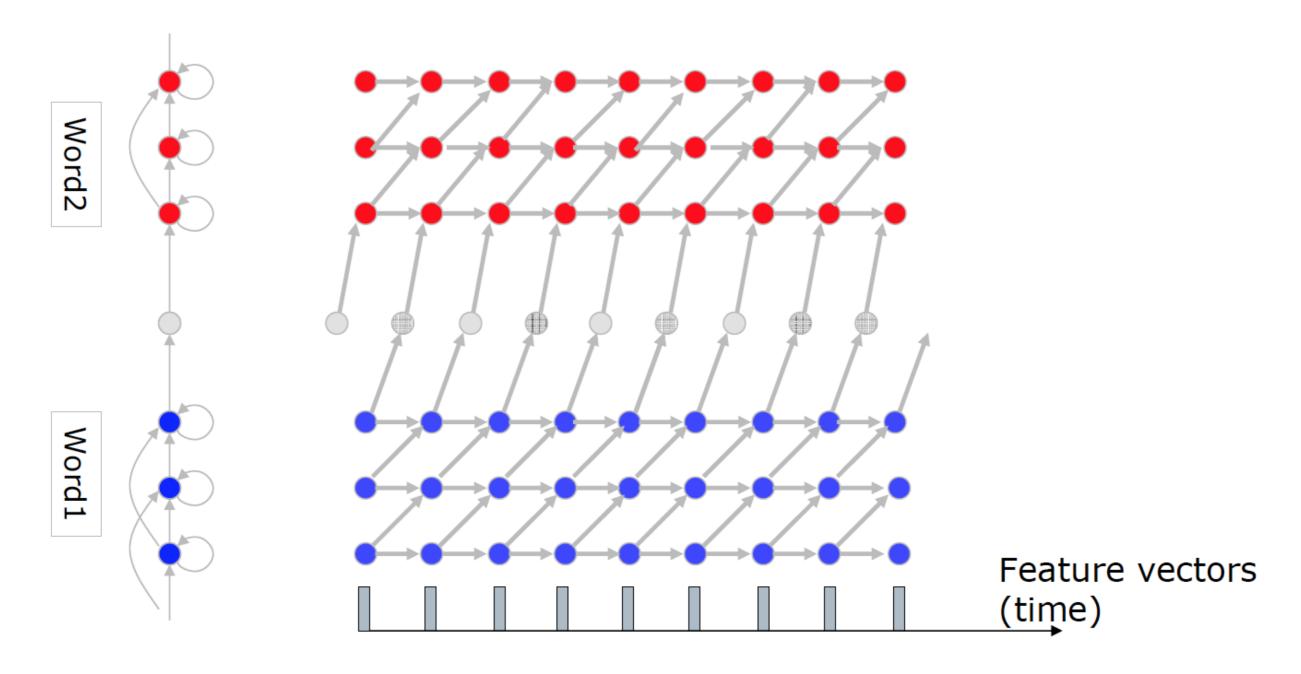
 Probability of reaching state N+1 at time t is equivalent to the probability of exiting word1 at time t

## Connecting Word HMMs



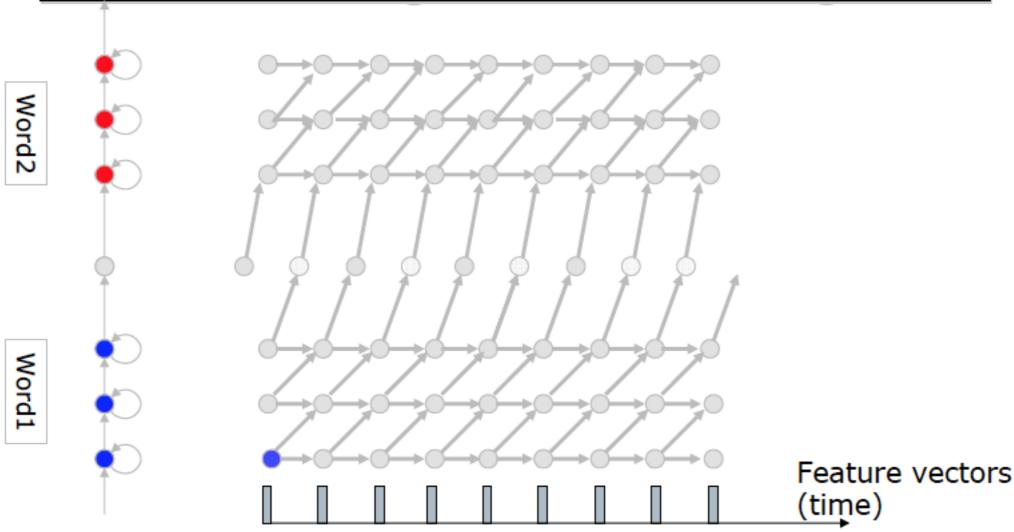


 One can also introduce the probability of word2 following word1 in this case.



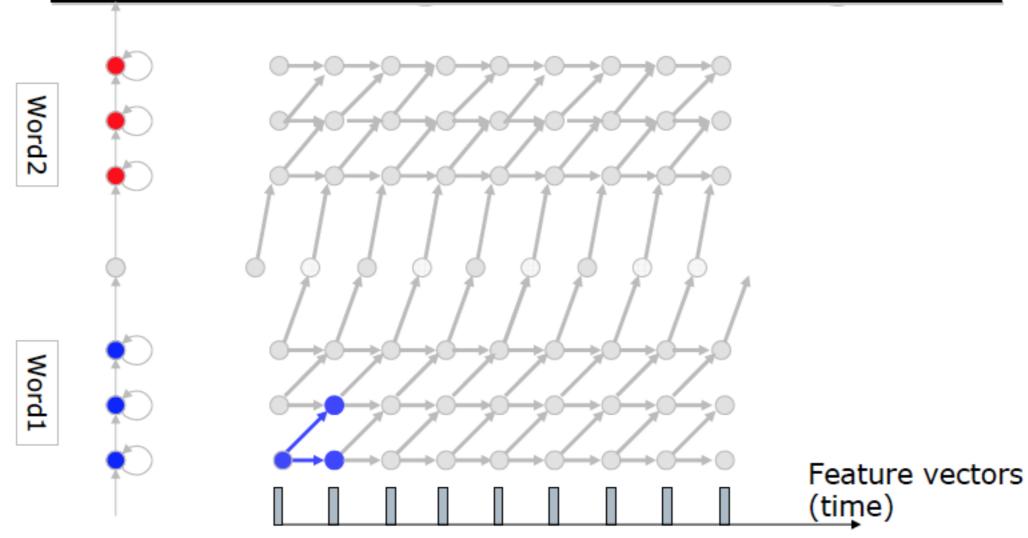
 Null states don't have time instant associated so they happen between two time instants.

#### Forward Through a non-emitting State



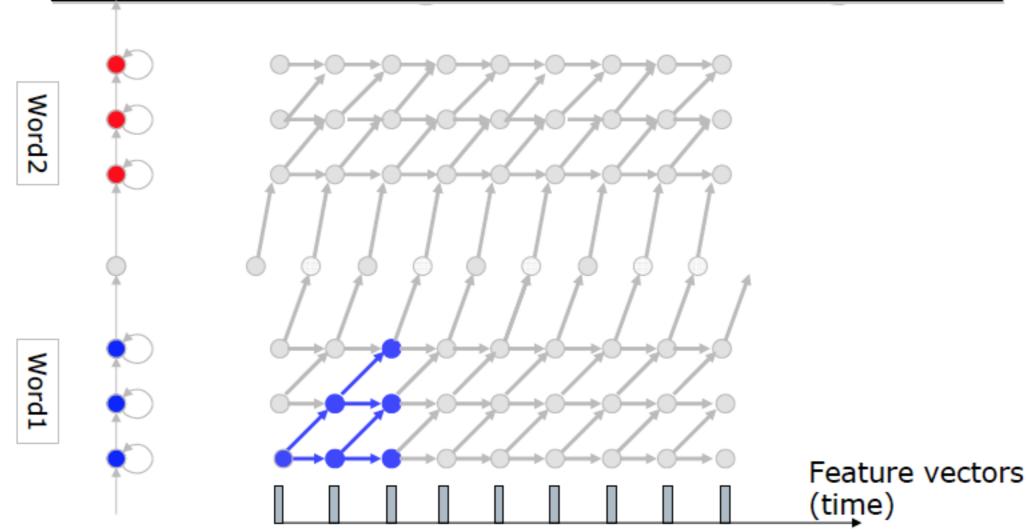
 At the first instant only one state has a non-zero forward probability

### Forward Through a non-emitting State



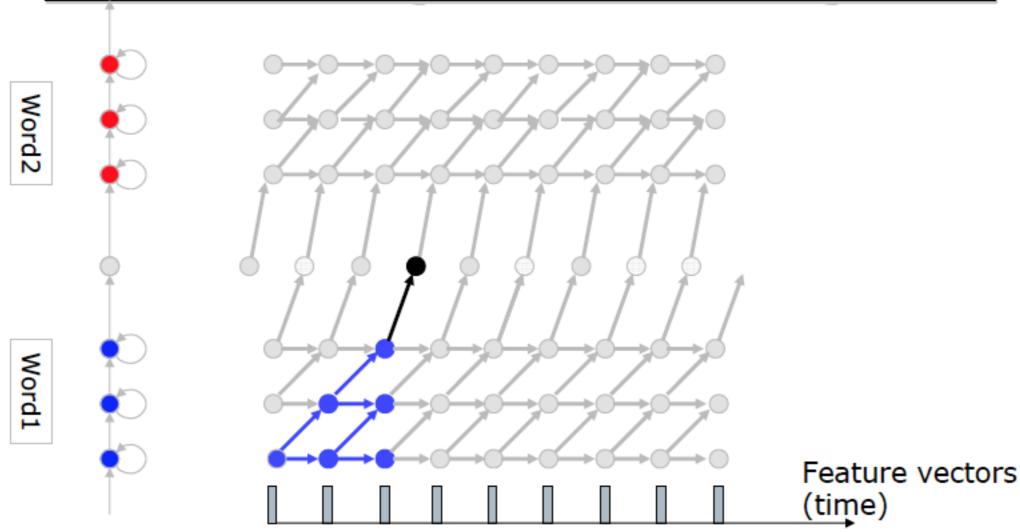
- From time 2 a number of states can have non-zero forward probabilities
  - Non-zero alphas

#### Forward Through a non-emitting State



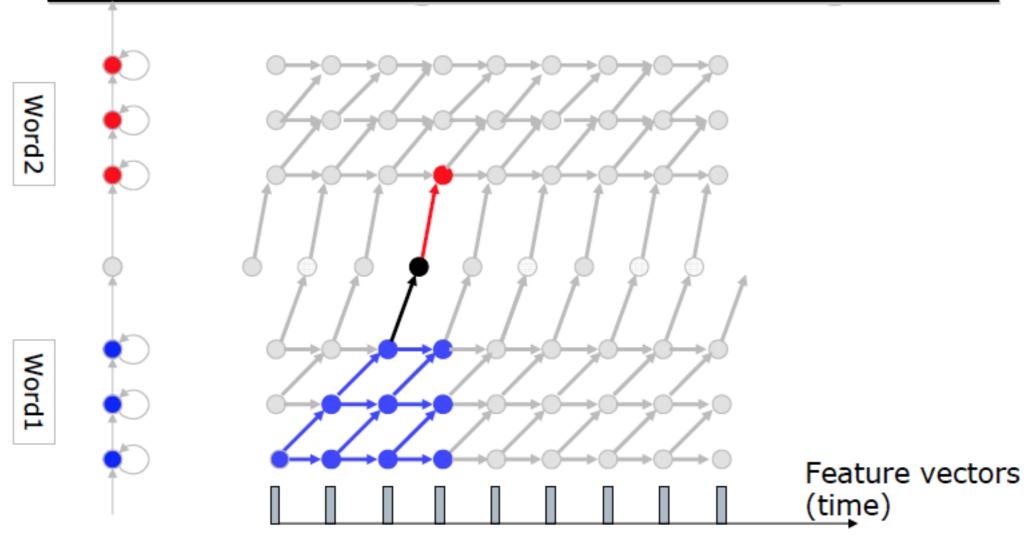
- From time 2 a number of states can have non-zero forward probabilities
  - Non-zero alphas

#### Forward Through a non-emitting State



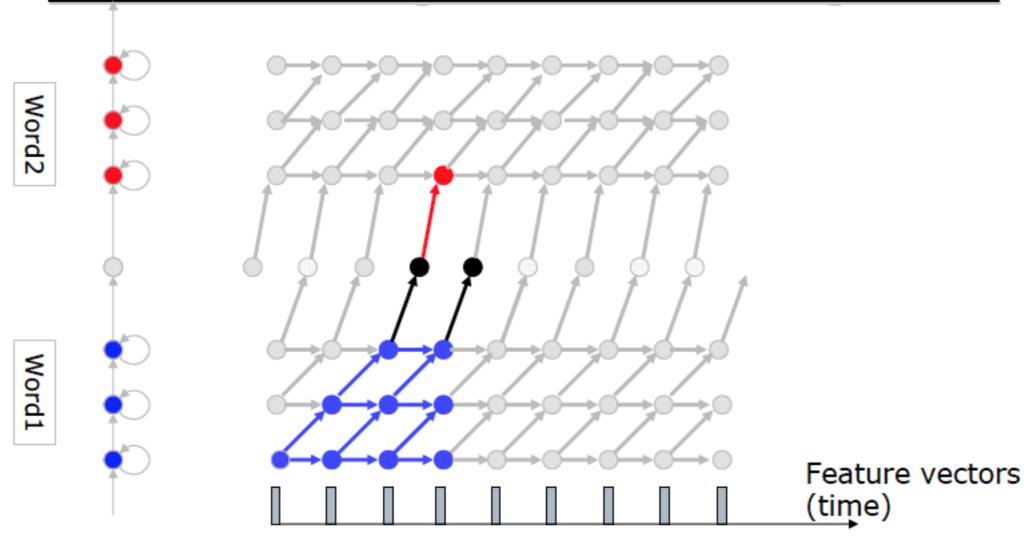
Between time 3 and time 4 (in this trellis) the non-emitting state gets a non-zero alpha

#### Forward Through a non-emitting State



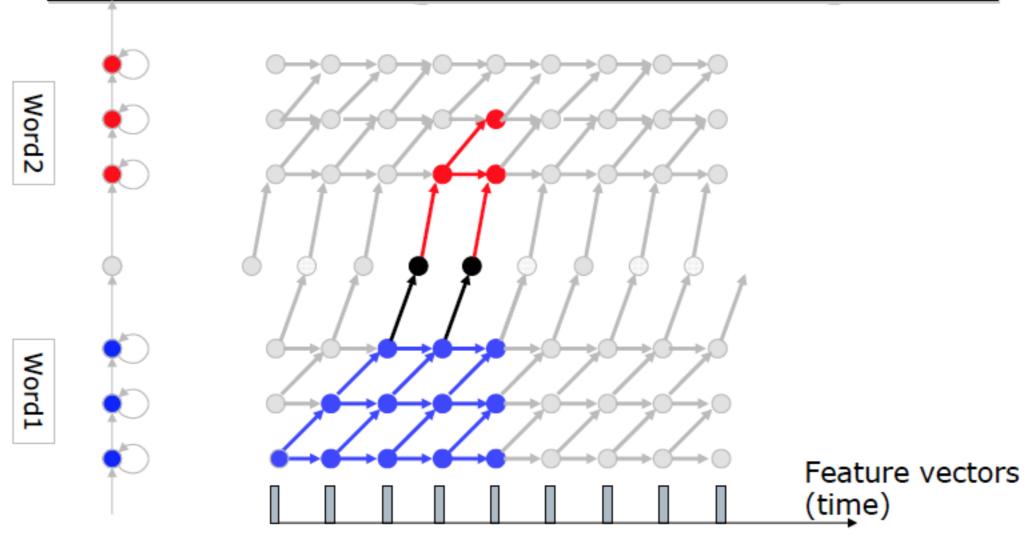
At time 4, the first state of word2 gets a probability contribution from the non-emitting state

#### Forward Through a non-emitting State



Between time4 and time5 the non-emitting state may be visited

#### Forward Through a non-emitting State



At time 5 (and thereafter) the first state of word 2 gets contributions both from an emitting state (itself at the previous instant) and the non-emitting state