

A Three-phase Inverter Based Overmodulation Strategy of Asymmetrical Six-phase Induction Machine

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Abstract—Overmodulation (OVM) techniques of asymmetrical six-phase induction machine (ASIM) achieve higher DC-bus utilization by applying non-zero average voltage in non-energy transfer plane. This results into unwanted current and associated copper loss. Existing OVM techniques reduce this loss with PWM techniques from six-dimensional space-vector perspective, which is both conceptually difficult and computationally challenging. The computational complexity of space-vector based techniques is reduced by few existing PWM techniques in linear region by splitting the six-phase inverter as two three-phase inverters and modulate these inverters with linear PWM techniques in 30 degree phase-shifted. This is referred as three-phase inverter based technique in the title. But, three-phase inverter based OVM technique doesn't exist in literature due to lack of knowledge of the relationship between six-dimensional space-vectors and space-vectors of two three-phase inverters. This paper first establishes this relation. Based on this relation, an OVM technique is proposed where the reference voltage vectors of two inverters are phase-shifted by 30 degree but of different magnitudes. The proposed technique achieves THD, WTHD performances similar to space-vector based best technique of ASIM with reduced computational complexity, as shown in details. The proposed strategy is validated through experiments and simulations on six-phase induction machine at 4.2 kW.

Index Terms—Multiphase machines, pulse width modulation (PWM), overmodulation, asymmetrical six-phase induction motor, six-phase drives, harmonic minimization.

I. INTRODUCTION

A SYMMETRICAL six-phase induction machine (ASIM) is one of the most common multi-phase machines which is preferred over conventional three-phase (3ϕ) machine in high power applications due to advantages, like, better fault tolerance for having phase redundancy, lower space-harmonic content, less susceptibility towards excitation harmonics, reduced rating of the power-electronic components compared to three-phase machine [1], [2]. Fig. 1 shows the six phase windings of ASIM where the six windings can be seen as two sets of balanced 3ϕ windings and the spatial shift between these two sets is 30° electrical. Due to this winding structure, 6^{th} , 18^{th} , $12k + 6$, $k = 0, 1, \dots$ order harmonic

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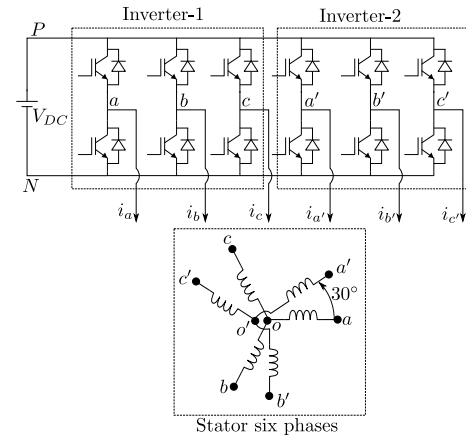


Fig. 1: Six-phase inverter fed ASIM

torques can't be produced even with non-sinusoidal excitation, [3]. This fact makes ASIM advantageous over symmetrical six-phase where all six windings are symmetrically displaced by 60° . Significant amount of research has been done over last few decades to thoroughly investigate ASIM modeling and design [4]–[6], PWM techniques [3], [6]–[17], different control strategies [18], [19], fault tolerant operation [20]–[22].

Dynamic model of ASIM had been derived in multiple ways in the existing literature, like, double d-q theory ([23]), vector-space decomposition (VSD) theory ([6]). The later one is the most popular way to model ASIM and the modulation strategy proposed in this paper is based on this theory. A 6×6 transformation matrix, T , was proposed in VSD theory to model the six-dimensional ($6 - D$) ASIM. T transforms $6 - D$ quantities from original frame to three $2 - D$ orthogonal subspaces. It is shown in [6] that one of the subspaces, namely $\alpha - \beta$, is solely responsible for electromagnetic energy transfer. The two $2 - D$ subspaces, which are not associated with energy transfer, have been labelled as $z_1 - z_2$ and $o_1 - o_2$ in this paper. The impedance of $z_1 - z_2$ and $o_1 - o_2$ subspaces are small as it consists of winding resistance and leakage inductance. As $\alpha - \beta$ is associated with energy transfer, this subspace should be excited with balanced fundamental voltage in order to generate ripple-free torque in ASIM. With the balanced winding with two isolated neutrals, as shown in Fig. 1, $o_1 - o_2$ plane can't be excited. Although applied voltage in $z_1 - z_2$ doesn't create torque ripple, it causes large circulating current and associated copper loss due to low impedance of the equivalent circuit.

Therefore, applied voltage in $z_1 - z_2$ should be as small as possible. Linear modulation techniques of ASIM don't inject any voltage in $z_1 - z_2$ whereas the overmodulation techniques do inject some voltage in $z_1 - z_2$ in order to extend the region in $\alpha - \beta$ plane available for modulation.

Linear PWM techniques of ASIM can be broadly classified into two categories: 1) Four-dimensional space-vector based PWM techniques, where at least four active vectors along with the zero vector are used within a sampling period in order to generate reference voltage vector in $\alpha - \beta$ and zero average voltage vector in $z_1 - z_2$ plane, [6]–[11]. Here, one technique differs from others in terms of current-ripple performances. 2) Two 3ϕ -inverter (TINV) based PWM techniques, as proposed by [15], [16], where the 6ϕ inverter is split into two 3ϕ inverters, inverter-1 and inverter-2 of Fig. 1. Inverter-1 is modulated with half of the reference voltage vector in $\alpha - \beta$ plane. Reference vector of inverter-2 is of same magnitude that of inverter-1 but phase-shifted by 30° . The second category of techniques, i.e. TINV techniques, are referred as three-phase inverter based technique in the title of this paper. Carrier-comparison based implementation of TINV techniques is computationally less burdened compared to implementation of space-vector based techniques where complex six-dimensional transformation is involved. Modulation index (M) is defined as the ratio of the peak of the fundamental component of the line to neutral voltage to the DC-bus voltage. The maximum M achievable by both of these categories of techniques is 0.577.

Overmodulation (OVM) of PWM converter is important for higher DC bust utilization and reduction in voltage rating of the switching devices and to meet higher voltage requirement during transients and peak loading conditions, [24]. OVM of multi-phase machines is interesting because unlike the three-phase case, there is no torque pulsation here. OVM of five phase machines has been widely discussed in literature, [25]–[29]. OVM techniques of ASIM extend the range of M upto 0.622 by applying non-zero average voltage in $z_1 - z_2$ plane. But, RMS voltage injected in this plane should be small in order to have small resultant current and associated copper loss. [3] and [10] proposed four-dimensional space-vector based OVM techniques. The proposed technique in [3], later named as *CSVPWM* in [6], uses two large active vectors adjacent to reference voltage vector in $\alpha - \beta$ and it injects substantial amount of harmonic voltages in $z_1 - z_2$ plane. To reduce this RMS voltage injection, [10] has proposed two OVM algorithms, which are labelled as *SVOVM4-1* and *SVOVM4-2*, respectively, in this paper. These techniques use two large and two medium active vectors adjacent to reference voltage vector in $\alpha - \beta$ plane. *SVOVM4-2* is the best known OVM technique in ASIM literature. One should notice that both five-phase machine with one isolated neutral and ASIM with two isolated neutrals require four-dimensional modulation. Four-dimensional space-vector based OVM techniques for five-phase machines are available in [25], [27], [28].

But, TINV based OVM techniques is an open problem in ASIM literature. Although [17] has applied standard 3ϕ OVM techniques, as proposed by [30], on individual 3ϕ inverters in 30° phase-shifted way, this technique injects harmonic

voltages in $\alpha - \beta$ which in turn causes low-frequency torque pulsations violating OVM condition of ASIM. In this paper, a TINV based OVM technique is proposed which can be implemented with reduced computational complexity, as TINV based linear techniques, and without compromising on the performance of the drive. Followings are the contributions of this paper.

- In order to analyse OVM from TINV perspective, a key relation between the six-phase quantities, obtained using the transformation defined in [6], with three-phase quantities, obtained through familiar $3\phi - 3\phi$ Clarke's transformation, is derived. This relationship is new to the literature.
- Using that relationship, a TINV based OVM technique of ASIM is proposed. This technique split the reference voltage vector in $\alpha - \beta$ plane into two reference voltage vectors of two 3ϕ inverters such that they are phase-shifted by 30° and of different magnitudes in OVM region but of same magnitude in linear region. The proposed strategy works upto $M = 0.5977$. It attains voltage THD performance almost similar to *SVOVM4-2*.
- A detailed comparison of computational burdens between the TINV based proposed technique and four-dimensional space-vector based *SVOVM4-2* shows that the proposed technique can be implemented at much reduced computational cost without involving complex six-dimensional transformation.

The organization of the paper is as follows: section II briefly discusses about the modeling of ASIM, III discusses the existing modulation techniques. The important relationships between three-phase and six-phase space-vectors and an OVM technique based on the above relationship are presented in sections IV and V respectively. Finally, the implementation strategy, simulation and experimental results and conclusions are discussed in sections VI, VII and VIII respectively.

II. DYNAMIC MODEL OF ASIM

$$X_i \triangleq \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\ 1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} X_j$$

$$X_i = X_{\alpha\beta z_1 z_2 o_1 o_2} = [x_\alpha \ x_\beta \ x_{z_1} \ x_{z_2} \ x_{o_1} \ x_{o_2}]^T$$

$$X_j = X_{abca'b'c'} = [x_a \ x_b \ x_c \ x_{a'} \ x_{b'} \ x_{c'}]^T \quad (1)$$

As six windings of ASIM can be excited from six independent voltage or current sources, ASIM is six-dimensional. A 6×6 orthonormal transformation matrix T , given in (1), has been used in [6] for analysing the dynamic model of ASIM. The co-ordinates in original and transformed domains are labeled as $a-b-c-a'-b'-c'$ and $\alpha-\beta-z_1-z_2-o_1-o_2$

respectively. Here X is a six-dimensional vector whose element x represents the machine variables, like voltage, current, flux-linkage etc.

A six-phase inverter fed squirrel-cage ASIM is shown in Fig. 1 where the six-phase inverter can be identified as two three-phase inverters, inverter-1 and inverter-2 respectively. The terminals of the six stator phase windings are a, b, c, a', b', c' , which are directly connected to the poles of the six legs of the inverter. These two sets of balanced three-phase windings are connected in star fashion and they form two isolated neutral points, o and o' . Therefore, voltages impressed on $o_1 - o_2$ plane by any of the switching state of this particular configuration will be zero (as the last two rows of T give the zero-sequence components of two sets of three-phase line-neutral voltages). So, discussions in the subsequent sections are focused to generate desired output voltages in $\alpha - \beta$ and $z_1 - z_2$ subspaces.

$$\begin{aligned}\vec{\psi}_{s,\alpha\beta} &= L_s \vec{i}_{s,\alpha\beta} + L_M e^{j\theta_r} \vec{i}_{r,\alpha\beta}; & \vec{\psi}_{s,z_1 z_2} &= L_{ls} \vec{i}_{s,z_1 z_2}; \\ \vec{\psi}_{r,\alpha\beta} &= L_M e^{-j\theta_r} \vec{i}_{s,\alpha\beta} + L_r \vec{i}_{r,\alpha\beta}; & \vec{\psi}_{r,z_1 z_2} &= L_{lr} \vec{i}_{r,z_1 z_2};\end{aligned}\quad (2)$$

$$\vec{v}_{k,\alpha\beta/z_1 z_2} = r_k \vec{i}_{k,\alpha\beta/z_1 z_2} + \frac{d}{dt} \vec{\psi}_{k,\alpha\beta/z_1 z_2}; \quad k = s, r \quad (3)$$

The machine model in $\alpha - \beta$ and $z_1 - z_2$ subspaces, as obtained by [6], are given in (2) and (3). Here, $\vec{\psi}_{s,\alpha\beta} = \psi_{s\alpha} + j\psi_{s\beta}$; $\psi_{s\alpha}$ and $\psi_{s\beta}$ are stator flux-linkages in α and β domains, respectively. Similarly, other quantities of (2) and (3) can be interpreted. $L_s - L_M = L_{ls}$; $L_r - L_M = L_{lr}$. Subscript s and r are used to denote stator and rotor quantities, respectively. Followings are few assumptions made during this modeling- sinusoidal winding distribution, infinite permeability of iron, linear and unsaturated magnetic material, uniform air-gap between stator and rotor. Three important properties of this model are outlined below.

- 1) Six subspaces in transformed domain are orthogonal to each other and can be grouped into three two-dimensional subspaces, viz., $\alpha - \beta$, $z_1 - z_2$ and $o_1 - o_2$ respectively. The dynamical equations of ASIM in these three subspaces can be analysed independently due to their decoupled nature.
- 2) The mutual coupling between the stator and rotor windings appear only in $\alpha - \beta$ subspace. From this, it can be concluded that only $\alpha - \beta$ subspace is associated with electromechanical energy conversion and torque production. Both the dynamical and steady-state equivalent circuits in this subspace are similar to the equivalent circuits of three-phase induction machine with 3ϕ Clarke's transformation.
- 3) The flux-linkages in $z_1 - z_2$ and $o_1 - o_2$ subspaces have no coupling between stator and rotor, as if air-gap flux doesn't appear in these subspaces. The equivalent circuits in these subspaces have winding resistance and leakage inductance. As the impedances of these equivalent circuits are small, any voltage applied in these subspaces causes large current resulting into copper losses and this large current doesn't contribute in electromagnetic energy conversion or torque pulsation.

From the above discussion it is clear that the excitation of $\alpha - \beta$ plane with balanced fundamental voltage will result into

operation of ASIM similar to 3ϕ induction machine without creating any torque ripple. The voltage excitations in $z_1 - z_2$ and $o_1 - o_2$ subspaces should be zero or as small as possible in order to drive ASIM efficiently. Although the sinusoidal winding distribution is an ideal assumption, which is not the case in practical, space-harmonics of non-sinusoidal winding distribution can be neglected as the mutual inductance due to n^{th} harmonic falls rapidly, as $\frac{1}{n^2}$, [31]. Incorporation of mutual leakage inductance in the dynamic modeling doesn't change the key properties as listed above from 1) to 3), [4].

III. EXISTING MODULATION TECHNIQUES

A brief review of existing modulation techniques of asymmetrical six-phase inverter drives is presented in this section. These modulation techniques can be broadly classified into two categories: linear modulation techniques and overmodulation techniques. To avoid the unnecessary copper loss, average voltage in $z_1 - z_2$ plane is kept zero in linear modulation region. In overmodulation region, fundamental voltage in $\alpha - \beta$ is increased at the cost of applying non-zero voltage in $z_1 - z_2$ plane. As no harmonic is injected in $\alpha - \beta$, there is no torque pulsation in overmodulation region. The main objective of the overmodulation techniques is to reduce RMS current in $z_1 - z_2$ plane to reduce the copper loss.

A. Linear Modulation Region

The modulation problem in linear region can be approached in two ways- two-inverter (TINV) and space-vector (SVPWM) approaches.

1) *TINV Techniques*: Let the controller provides \bar{v}_α and \bar{v}_β , average reference voltages in $\alpha - \beta$ plane or reference voltage vector in $\alpha - \beta$ (\vec{V}_{ref}) which is given by (4).

$$\vec{V}_{ref} \triangleq \bar{v}_\alpha + j\bar{v}_\beta \quad (4)$$

In linear range $\bar{v}_{z_1} = \bar{v}_{z_2} = 0$. Here bar represents the average voltage over a switching cycle. Due to isolated neutral connections, $\bar{v}_{o_1} = \bar{v}_{o_2} = 0$. By taking inverse transformation (T^{-1}) on these transformed domain voltages, the average line-neutral voltages \bar{v}_{io} and $\bar{v}_{i'o'}$, $i = a, b, c$, are obtained. Based on these two sets of three-phase line-neutral voltages, two common-mode voltages \bar{v}_{oN} and $\bar{v}_{o'N}$ are chosen independently, N is DC-bus negative terminal as shown in Fig. 1. The most popular choice of common-mode voltage is $\frac{1}{2} + \frac{mid}{2}$ where mid is the middle value of the corresponding three average line-neutral voltages.

For balanced sinusoidal operation in steady-state, suppose $\bar{v}_\alpha = \sqrt{3}V_o \cos(\omega_o t = \theta)$, $\bar{v}_\beta = \sqrt{3}V_o \sin \theta$ and $\bar{v}_{z_1} = \bar{v}_{z_2} = \bar{v}_{o_1} = \bar{v}_{o_2} = 0$. Therefore,

$$\vec{V}_{ref} = \sqrt{3}V_o e^{j\theta} \quad (5)$$

$$\bar{v}_{io} = V_o \cos\left(\theta - \frac{k_i 2\pi}{3}\right), \bar{v}_{i'o'} = V_o \cos\left(\theta - \left(\frac{\pi}{6} + \frac{k_i 2\pi}{3}\right)\right) \quad (6)$$

Applying T^{-1} on these transformed domain voltages, line-neutral voltages can be obtained as (6), where $i \in \{a, b, c\}$

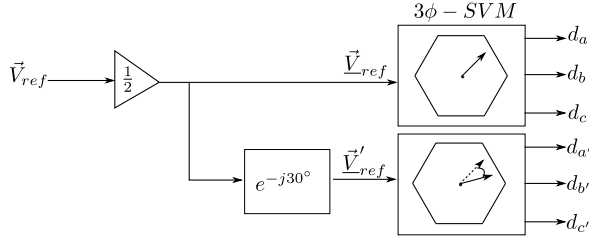


Fig. 2: Modulation of ASIM in linear region based on TINV strategy

and $k_a = 0, k_b = 1, k_c = 2$. These two sets are three-phase balanced sinusoidal voltages and they are phase-shifted by 30° to each other. Equation (7a) defines 3ϕ Clarke transformation, where C relates $a - b - c$ to $\underline{\alpha} - \underline{\beta} - \underline{o}$. One should note here that $\underline{\alpha} - \underline{\beta}$ of 3ϕ is different from $\underline{\alpha} - \underline{\beta}$ of 6ϕ , underline is used to denote the three-phase transformed domain quantities. The reference voltage vectors of the two three-phase inverters are commonly defined as (8), where prime denotes the quantities related to second inverter. Plugging the balanced sinusoidal average line-neutral voltages of (6) in (7), the reference voltage vectors of the two inverters are obtained from the definition given in (8) as $\underline{\vec{V}}_{ref} = \frac{\sqrt{3}}{2} V_o e^{j\theta}$ and $\underline{\vec{V}}'_{ref} = \frac{\sqrt{3}}{2} V_o e^{j(\theta-30^\circ)}$. One can also see that for balanced sinusoidal operation, $\underline{\vec{V}}_{ref} = \frac{\underline{\vec{V}}_{ref}}{2}$ and $\underline{\vec{V}}'_{ref} = \frac{\underline{\vec{V}}_{ref}}{2} e^{-j30^\circ}$.

$$\underbrace{\begin{bmatrix} v_{\underline{\alpha}} \\ v_{\underline{\beta}} \\ v_{\underline{o}} \end{bmatrix}}_{V_{\underline{\alpha}\underline{\beta}\underline{o}}} \triangleq \frac{1}{\sqrt{3}} \underbrace{\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ 1 & 1 & 1 \end{bmatrix}}_C \underbrace{\begin{bmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{bmatrix}}_{V_{abc}} \quad (7a)$$

$$V_{\underline{\alpha}'\underline{\beta}'\underline{o}'} \triangleq CV_{a'b'c'}; \quad V_{a'b'c'} = [v_{a'o'} \quad v_{b'o'} \quad v_{c'o'}]^T \quad (7b)$$

$$\underline{\vec{V}}_{ref} \triangleq \underline{\bar{v}}_{\underline{\alpha}} + j\underline{\bar{v}}_{\underline{\beta}}; \quad \underline{\vec{V}}'_{ref} \triangleq \underline{\bar{v}}_{\underline{\alpha}'} + j\underline{\bar{v}}_{\underline{\beta}'} \quad (8)$$

Therefore, it can be seen that modulation of ASIM in linear range can be performed from two three-phase inverters with reference voltage vectors of equal magnitude but phase-shifted by 30° and reference voltage vector of one of the inverters is half of the reference voltage vector in $\alpha - \beta$ of 6ϕ machine. This is illustrated in Fig. 2 where d_i represents the duty ratio of the top switch of i^{th} leg. Here transformation matrix T

isn't required. Based on the above idea, modulation techniques like, *Double zero-sequence injection technique* ([16]), *Three-waves based zero-sequence injection PWM*, *Six-waves based zero-sequence injection PWM* ([12]) have been proposed. If modulation index (M) is defined as the ratio of peak line-neutral fundamental voltage and DC-bus voltage, i.e., $M = \frac{V_o}{V_{DC}}$, maximum M of asymmetrical six-phase drive in linear region is 0.577.

2) *SVPWM Techniques*: In this technique, voltages generated by 64 switching states of the six-phase inverter are mapped into $\alpha - \beta$ and $z_1 - z_2$ subspaces using T . This mapping results into 60 active vectors and 4 zero vectors in both $\alpha - \beta$ and $z_1 - z_2$ subspaces ([6], [12], [17]). Using these switching vectors, the desired reference voltage vectors in these subspaces are synthesized. In linear region, reference voltage vectors in $\alpha - \beta$ is given by (5) and (9) gives the reference voltages in $z_1 - z_2$.

$$\underline{\bar{v}}_{z_1} + j\underline{\bar{v}}_{z_2} = 0 \quad (9)$$

To satisfy the four constraints of (5) and (9), minimum four active states have to be applied along with zero states. These 4 active vectors can be chosen in multiple ways. Depending upon the choice of vectors, different modulation techniques exist in the literature like, *Vector-space decomposition (VSD)* technique in [6], *24-sector vector-space decomposition* in [8], *modified 24-sector vector-space decomposition* in [11], [13], *four vector technique* with 2 large and 2 medium vectors in [9], [10]. It is possible to show that all of these techniques can attain maximum M equal to 0.577 in linear region.

It is clear from the above discussion that *with TINV techniques one can modulate ASIM from the knowledge of two-level three-phase VSI without involving complicated six-dimensional transformation, T*. But unlike TINV, SVPWM techniques require knowledge of T and this makes it both conceptually and computationally challenging compared to TINV techniques.

B. Overmodulation Region

As $M > 0.577$ in overmodulation region, the six-phase inverter will inject harmonics in line-neutral voltages which appear only in $z_1 - z_2$ plane but not in $\alpha - \beta$. Therefore, at the outset $\underline{\bar{v}}_{\underline{\alpha}}$ and $\underline{\bar{v}}_{\underline{\beta}}$ are known as before (coming from the controller), but $\underline{\bar{v}}_{z_1}$, $\underline{\bar{v}}_{z_2}$ are not known. Therefore, all of the overmodulation techniques try to define $\underline{\bar{v}}_{z_1}$ and $\underline{\bar{v}}_{z_2}$ by adopting different strategies.

$$v_{\alpha} = \frac{1}{\sqrt{3}} \left((v_{ao} - \frac{v_{bo}}{2} - \frac{v_{co}}{2}) + \frac{\sqrt{3}}{2} (v_{a'o'} - v_{b'o'}) \right) = \left(v_{\underline{\alpha}} + \frac{1}{2} (v_{a'o'} - v_{b'o'}) \right) \quad (10)$$

$$v_{\beta} = \frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}}{2} (v_{bo} - v_{co}) - (v_{c'o'} - \frac{v_{b'o'}}{2} - \frac{v_{a'o'}}{2}) \right) = \left(v_{\underline{\beta}} - \frac{1}{\sqrt{3}} (v_{c'o'} - \frac{v_{b'o'}}{2} - \frac{v_{a'o'}}{2}) \right) \quad (11)$$

$$v_{\alpha} + jv_{\beta} = \left(v_{\underline{\alpha}} + jv_{\underline{\beta}} \right) + \underbrace{\frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}}{2} (v_{a'o'} - v_{b'o'}) - j(v_{c'o'} - \frac{v_{b'o'}}{2} - \frac{v_{a'o'}}{2}) \right)}_{\underline{V}_u} \quad (12)$$

$$\underline{V}_u = \left((v_{a'o'} - \frac{v_{b'o'}}{2} - \frac{v_{c'o'}}{2}) + j\frac{\sqrt{3}}{2} (v_{b'o'} - v_{c'o'}) \right) \left(\frac{\sqrt{3}}{2} + j\frac{1}{2} \right) = \sqrt{3} (v_{\underline{\alpha}'} + jv_{\underline{\beta}'}) e^{j30^\circ} \quad (13)$$

1) *SVPWM Techniques*: [3] proposed a strategy (which later named as *CSVPWM* in [6]) that uses two largest active vectors adjacent to the reference voltage vector in $\alpha - \beta$ plane in order to satisfy (5). Although this technique attains maximum M up to 0.622, it applies significant amount of voltage in $z_1 - z_2$ plane. To reduce this applied voltage, [10] has proposed two overmodulation algorithms, which are termed as *SVOVM4-1 (space-vector based overmodulation technique-1 with 4 active vectors)* and *SVOVM4-2* in this paper, with the following assumptions:

- 1) These techniques use 4 active vectors. These 4 vectors form two sets of vectors 30° apart in $\alpha - \beta$ plane. Each set comprising of 1 large and 1 medium active vectors of $\alpha - \beta$ plane which are along the same direction in $\alpha - \beta$ and 180° apart in $z_1 - z_2$.
- 2) No zero vector is applied.

With the above choice of vectors, *SVOVM4-2* has optimized the length of the applied average voltage vector in $z_1 - z_2$ plane.

2) *TINV Techniques*: [17] has extended the idea of two-inverter based modulation by operating individual three-phase inverters through overmodulation schemes proposed in [30] where the second 3ϕ inverter follows the first inverter at a phase lag of 30° with respect to fundamental frequency. But these overmodulation schemes inject harmonics in $\alpha - \beta$ plane and results into torque pulsation. Therefore, *no two-inverter based overmodulation technique exists in the literature*.

To suggest an overmodulation strategy based on two-inverter approach, one needs to relate the six-phase transformed domain quantities with three-phase transformed domain quantities. We can see from (8) that for *sinusoidal operation in linear range*, $\bar{v}_\alpha + j\bar{v}_\beta \triangleq \bar{V}_{ref} = \bar{V}'_{ref} + \bar{V}'_{ref}e^{j30^\circ}$.

- Does this relation hold in general?
- Can we write the reference voltage vector in $z_1 - z_2$ plane in terms of $\bar{v}_\alpha + j\bar{v}_\beta$ and $\bar{v}_{\alpha'} + j\bar{v}_{\beta'}$?

These questions are answered in the next section by proposing an alternative way of modeling the asymmetrical six-phase inverter.

IV. RELATIONSHIP BETWEEN THREE-PHASE AND SIX-PHASE SPACE-VECTORS

In (7a) and (7b), each of the two sets of three-phase voltages have been transformed individually to $\underline{\alpha\beta 0}$ and $\underline{\alpha'\beta'0'}$ using well-known Clarke's $3\phi - 3\phi$ transformation C . Now we will try to relate $V_{\alpha\beta z_1 z_2 o_1 o_2}$ with $V_{\alpha\beta 0}$ and $V_{\alpha'\beta'0'}$. Using (1), (7a) and (7b) for voltage quantities, we can write (10) and (11). Then combining (10) and (11), we get (12). The underbracketed term in RHS of (12), \mathbb{V}_u , is simplified and expressed compactly in (13). Using (12) and (13), one gets the key expression (14a). Similarly, combining v_{z_1} and v_{z_2} using (1), (7a), (7b) and (13), one gets another important expression (14b). Here * is complex conjugate operation. In (14c), $o_1 - o_2$ quantities have been written in terms of zero-sequence of the two three-phase inverters.

$$v_\alpha + jv_\beta = \left((v_{\underline{\alpha}} + jv_{\underline{\beta}}) + (v_{\underline{\alpha}'} + jv_{\underline{\beta}'})e^{j30^\circ} \right) \quad (14a)$$

$$v_{z_1} + jv_{z_2} = \left((v_{\underline{\alpha}} + jv_{\underline{\beta}}) - (v_{\underline{\alpha}'} + jv_{\underline{\beta}'})e^{j30^\circ} \right)^* \quad (14b)$$

$$v_{o_1} + jv_{o_2} = v_{\underline{o}} + jv_{\underline{o}'} \quad (14c)$$

Although the above relationship is shown for line-neutral voltages, it is true for other variables as well, like, current, flux linkage, etc. One can model ASIM with these 3ϕ space-vectors and apply (14) to obtain the model in $\alpha - \beta$ and $z_1 - z_2$. The resultant model is same as the model given in (2), (3), as obtained by [6]. Fig. 3 shows the two ways to obtain the transformed quantities from the quantities in original domain.

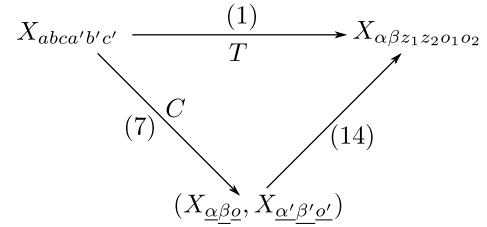


Fig. 3: Two ways to obtain quantities in transformed domains

We had anticipated (14a) in previous section for average vectors in linear range. But here we can see that both (14a) and (14b) are true on an instantaneous basis. These two expressions are the key results where the complex vectors in $\alpha - \beta$ and $z_1 - z_2$ are expressed as the linear combination of 3ϕ space-vectors and their complex conjugates. These are essential for analysis of overmodulation of six-phase drive from two-inverter perspective, as we will see in next section.

Two-inverter based linear modulation techniques can be readily explained using above two expressions. If we take switching cycle average of (14a) and (14b), then combining these equations with average reference voltage vector equations of linear modulation techniques ((4), (8) and (9)), one can get $\bar{V}_{ref} = \frac{\bar{V}'_{ref}}{2}$ and $\bar{V}'_{ref} = \frac{\bar{V}_{ref}}{2}e^{-j30^\circ}$.

V. PROPOSED OVERMODULATION TECHNIQUE

For the sake of convenience, two complex space-vectors are defined as follows.

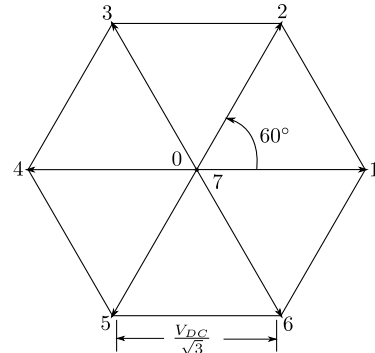


Fig. 4: Voltage vectors of three-phase inverter

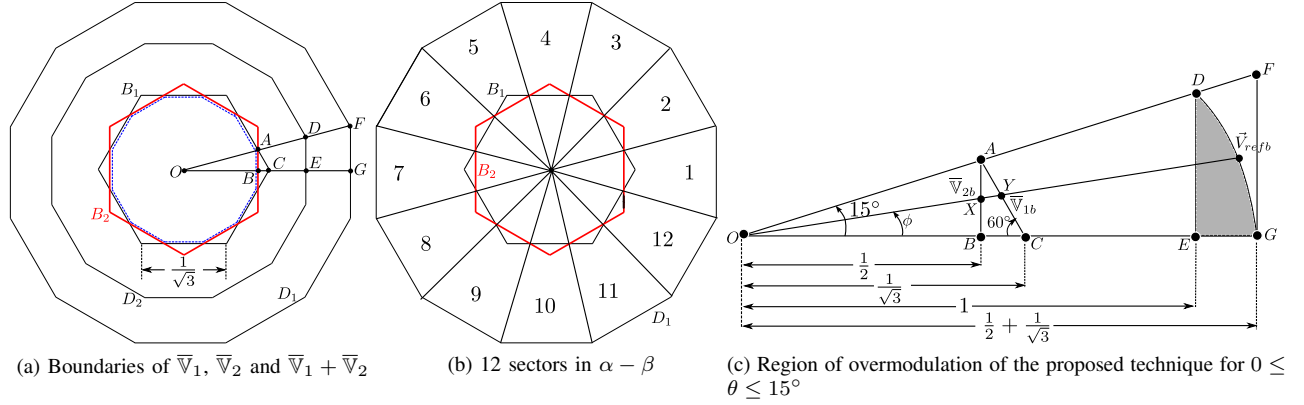


Fig. 5: Proposed overmodulation technique

$$\bar{V}_1 \triangleq (v_{\underline{\alpha}} + jv_{\underline{\beta}}); \bar{V}_2 \triangleq (v_{\underline{\alpha}'} + jv_{\underline{\beta}'}) e^{j30^\circ}$$

Taking switching cycle average on above equation and comparing with (8), one can see that $\bar{V}_1 = \vec{V}_{ref}$ and $\bar{V}_2 = \vec{V}'_{ref} e^{j30^\circ}$. With these new notations, reference voltage vectors in $\alpha - \beta$ and $z_1 - z_2$ can be written as follows.

$$\vec{V}_{ref} = (\bar{V}_1 + \bar{V}_2) \quad (15a)$$

$$\bar{v}_{z_1} + j\bar{v}_{z_2} = (\bar{V}_1 - \bar{V}_2)^* \quad (15b)$$

Each three-phase inverter of Fig. 1 has 8 permissible switching states. Fig. 4 shows the voltage vectors generated by these switching states in $\underline{\alpha} - \underline{\beta}$ (following (7a) or (7b)). Table I shows the labels used to identify the switching state of three-phase inverter. Each of these labels consists of three binary numbers corresponding to switching state of three legs, a , b and c , respectively, in order. Here 1 denotes the top switch of a leg is 'on' and bottom switch is 'off' and 0 denotes vice-versa. The tips of 6 voltage vectors lie on the vertices of a regular hexagon and two states result into the zero vector, as shown in Fig. 4. It is possible to show that the tip of the average voltage space-vectors generated by individual three-phase inverters (\vec{V}_{ref} and \vec{V}'_{ref}) lie within this hexagon in order to satisfy the duty ratio constraints of the switches.

TABLE I: Switching States of Three-phase Inverter

Switching State	Label	Switching State	Label
000	0	011	4
100	1	001	5
110	2	101	6
010	3	111	7

As $\bar{V}_1 = \vec{V}_{ref}$, \bar{V}_1 will lie within hexagonal boundary as \vec{V}_{ref} and the boundary is shown in Fig. 5a with the label B_1 . $\bar{V}_2 = \vec{V}'_{ref} e^{j30^\circ}$, therefore boundary of \bar{V}_2 , B_2 , is obtained by rotating the hexagonal boundary of \vec{V}'_{ref} by 30° , as shown in Fig. 5a. The length of the sides of these two hexagons is $\frac{V_{DC}}{\sqrt{3}}$. For brevity, all the lengths are scaled with a factor of $\frac{1}{V_{DC}}$.

Therefore, length of each side of these two hexagons becomes $\frac{1}{\sqrt{3}}$, as shown in Fig. 5a. The outermost do-decagon D_1 in Fig. 5a is the largest do-decagon obtained from vector addition of all possible \bar{V}_1 and \bar{V}_2 lying within B_1 and B_2 respectively. Therefore, \vec{V}_{ref} should lie within D_1 in accordance to (15a). In linear modulation range, both (15b) and (9) give $\bar{V}_1 = \bar{V}_2$. With this constraint, the maximum boundary of both \bar{V}_1 and \bar{V}_2 is the boundary of the intersection region of B_1 and B_2 , which has a shape of regular do-decagon. This is shown in Fig. 5a by dotted blue line. D_2 is obtained by doubling this intersection region and hence tip of \vec{V}_{ref} will lie within D_2 in linear range. In overmodulation ($M > 0.577$), tip of \vec{V}_{ref} will lie partially or fully in the region between D_1 and D_2 .

The $\alpha - \beta$ plane consists of 12 equivalent sectors, as shown in Fig. 5b. As the modulation in all of these 12 sectors will be similar, only sector-1 ($-15^\circ \leq \theta \leq 15^\circ$) is considered for discussion. The boundaries of \bar{V}_1 , \bar{V}_2 and $\bar{V}_1 + \bar{V}_2$ in 0° to 15° and 0° to -15° are symmetric with respect to real axis and therefore discussion can be further restricted to 0° to 15° . Fig. 5c shows the zoomed version of part of Fig. 5a when $0^\circ \leq \theta \leq 15^\circ$. $OC = \frac{1}{\sqrt{3}}$, as C is the vertex of hexagon B_1 . $\angle OCA = 60^\circ$, $\angle COA = 15^\circ$, $\therefore \angle OAC = 105^\circ$. Applying sine rule, we get

$$\frac{OC}{\sin \angle OAC} = \frac{OA}{\sin \angle OCA} \implies OA = \frac{1}{2 \cos 15^\circ}$$

As $\angle OBA = 90^\circ$, $OB = OA \cos 15^\circ = \frac{1}{2}$. $OE = 2 \times OB = 1$, as DE , part of D_2 , is obtained by doubling each vector lying on the line AB . These lengths are indicated in Fig. 5c. All variables in modulo sector (sector-1) will be labelled with subscript m .

In Fig. 5c, $\bar{V}_{1m} \in \Delta OAC$ and $\bar{V}_{2m} \in \Delta OAB$, where line segments AC and AB belong to the boundaries B_1 and B_2 respectively. Let $\bar{V}_{1m} = \rho_1 e^{j\phi_1}$ and $\bar{V}_{2m} = \rho_2 e^{j\phi_2}$. In linear modulation range, $\bar{V}_{1m} = \bar{V}_{2m}$ implies $\rho_1 = \rho_2$ and $\phi_1 = \phi_2$. As the maximum abscissa \bar{V}_{2m} can have is $OB = \frac{1}{2}$, therefore, the maximum abscissa \bar{V}_{1m} can have is also $OB = \frac{1}{2}$ in case of $\bar{V}_{1m} = \bar{V}_{2m}$ which implies $\bar{V}_{1m} + \bar{V}_{2m}$ can have maximum abscissa as $OE = 1$. Thus, linear techniques allow us to realize any $\vec{V}_{ref,m}$ whose tip lies within ΔODE , where

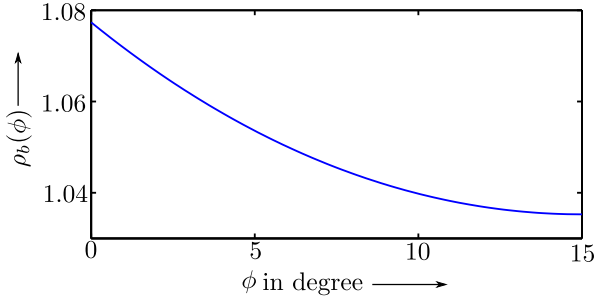


Fig. 6: $\rho_b(\phi)$ as function of ϕ with $V_{DC} = 1$

DE is part of D_2 . To realize any $\vec{V}_{ref,m}$ so that its tip lies within quadrilateral $DEGF$, where abscissa is greater than 1, we can't have both $\rho_1 = \rho_2$ and $\phi_1 = \phi_2$. So, let us keep $\phi_1 = \phi_2$ but $\rho_1 \neq \rho_2$. The area within quadrilateral $DEGF$, which can be utilized for modulation with this strategy, is determined as follows.

As $\phi_1 = \phi_2$, both \bar{V}_{1m} and \bar{V}_{2m} subtend same angle with respect to OG and suppose $\phi_1 = \phi_2 = \phi$. To find the maximum boundary achievable with this condition, let both \bar{V}_{1m} and \bar{V}_{2m} move along their boundary keeping $\phi_1 = \phi_2 = \phi$. These vectors (\bar{V}_{1b} and \bar{V}_{2b}) are shown in Fig. 5c. Subscript b stands for boundary value. In Fig. 5c, $\bar{V}_{1b} = OY$ and $\bar{V}_{2b} = OX$. $\angle COY = \phi = \angle BOX$. $\therefore \angle OYC = 120^\circ - \phi$. Applying sine rule, we get

$$\frac{OC}{\sin \angle OYC} = \frac{OY}{\sin \angle OCY} \text{ and } OX = \frac{OB}{\cos \phi}$$

and therefore, the absolute values of \bar{V}_{1b} and \bar{V}_{2b} can be expressed as functions of ϕ as follows.

$$\begin{aligned} OY = \rho_{1b}(\phi) &= \frac{1}{2 \cos(30^\circ - \phi)} \\ OX = \rho_{2b}(\phi) &= \frac{OB}{\cos \phi} = \frac{1}{2 \cos \phi} \end{aligned} \quad (16)$$

Using (15a), the boundary of $\vec{V}_{ref,m}$ (\vec{V}_{refb}) in this strategy can be determined by taking summation of $\rho_{1b}(\phi)$ and $\rho_{2b}(\phi)$ for $0^\circ \leq \phi \leq 15^\circ$. This is shown in Fig. 5c where $\vec{V}_{refb}(\phi) = \rho_b(\phi)e^{j\phi}$ moves along curve GD and $\rho_b(\phi) = \rho_{1b}(\phi) + \rho_{2b}(\phi)$. Therefore, in addition to $\triangle ODE$, the shaded region within $DEGF$ (DEG) can be used for modulation of ASIM in $\alpha - \beta$ plane. Clearly, $\rho_b(\phi = 0^\circ) = OG$ and $\rho_b(\phi = 15^\circ) = OD$. Fig. 6 plots $\rho_b(\phi)$ as function of ϕ for $0^\circ \leq \phi \leq 15^\circ$. It can be seen that $\rho_b(\phi)$ is monotonically decreasing function in this range and therefore OD is the minimum vector length among the vectors whose tips lie on the curve DG . Therefore, the maximum length of the voltage vector that is possible to synthesize for all possible angles ($0^\circ \leq \theta_m \leq 15^\circ$), is OD and the maximum modulation index attainable with this strategy can be obtained by equating the absolute value of the circular reference voltage vector of (5) with OD . As $V_o = MV_{DC}$ and all lengths are scaled with a factor $\frac{1}{V_{DC}}$, reference voltage vector of (5) in modulo sector becomes $\vec{V}_{ref,m} = \sqrt{3}Me^{j\theta_m}$, where θ_m is the angle in modulo sector. Therefore, the maximum modulation index,

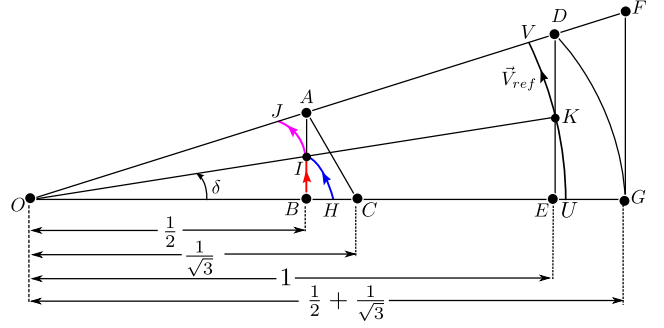


Fig. 7: \bar{V}_1 and \bar{V}_2 in the proposed technique for $0^\circ \leq \theta \leq 15^\circ$ and an $M \in (0.577, 0.5977)$

$M_{max} = \frac{\rho_b(15^\circ)}{\sqrt{3}} = \frac{1}{\sqrt{3} \cos 15^\circ} = 0.5977$, can be derived analytically.

A. Modulation Strategy

When M lies within the range $0 \leq M \leq 0.577$, tip of $\vec{V}_{ref,m}$ always lie within $\triangle ODE$ and therefore TINV based linear modulation strategy, as discussed in section III-A, will be adopted. For $0.577 \leq M \leq 0.5977$, tip of $\vec{V}_{ref,m}$ lies partially outside $\triangle ODE$ but within area ODG , as shown by curve UV in Fig. 7. Let, $\vec{V}_{ref,m}$ intersects the line DE at K at an angle δ and therefore $\sqrt{3}M \cos \delta = OE = 2OB = 1$, which gives

$$\delta = \cos^{-1} \left(\frac{1}{\sqrt{3}M} \right) \quad (17)$$

So, δ is function of M alone. For $0^\circ \leq \theta_m \leq \delta$, $\vec{V}_{ref,m}$ (UK) lies outside $\triangle ODE$. To satisfy (15a) with the adopted strategy ($\phi_1 = \phi_2$), following equation needs to be solved.

$$\bar{V}_{1m} + \bar{V}_{2m} \triangleq \rho_1 e^{j\phi_1} + \rho_2 e^{j\phi_2} = \vec{V}_{ref,m} = \sqrt{3}M e^{j\theta_m}$$

which gives

$$\phi_1 = \theta_m = \phi_2 \quad (18a)$$

$$\rho_1 + \rho_2 = \sqrt{3}M \quad (18b)$$

There are 4 unknowns and 3 equalities in the above equation. Therefore, we have one degree of freedom. It can be seen from (15b) that the vector length applied in $z_1 - z_2$ plane can be minimized by minimizing $|\bar{V}_1 - \bar{V}_2|$. Therefore, the above degree of freedom can be utilized in order to choose $\rho_{1,2}$ for minimizing $|\bar{V}_{1m} - \bar{V}_{2m}|$. With $\phi_1 = \phi_2$, minimization of $|\bar{V}_{1m} - \bar{V}_{2m}|$ becomes equivalent to minimization of $|\rho_1 - \rho_2|$. To minimize $|\rho_1 - \rho_2|$, \bar{V}_{1m} and \bar{V}_{2m} should be as close as possible. When $\bar{V}_{1m} = \bar{V}_{2m}$ and both move along BA , we get ED in $\alpha - \beta$ plane. Therefore, to attain voltage vector beyond ED keeping $|\rho_1 - \rho_2|$ minimum, \bar{V}_{2m} should lie on its boundary, i.e., BA . Hence for $0^\circ \leq \theta_m \leq \delta$ (UK part), \bar{V}_{2m} moves along BI (locus is shown by red colour) which gives $\rho_2 = \frac{OB}{\cos \phi_2} = \frac{1}{2 \cos \theta_m}$. To satisfy (18b), $\rho_1 = (\sqrt{3}M - \frac{1}{2 \cos \theta_m})$ and \bar{V}_{1m} moves along HI in Fig.

7. In KV part, $\vec{V}_{ref,m}$ lies within $\triangle ADE$ and $\rho_1 = \rho_2$ will give zero voltage injection in $z_1 - z_2$ plane. Therefore for $\delta \leq \theta_m \leq 15^\circ$, $\bar{V}_{1m} = \bar{V}_{2m} = \frac{\bar{V}_{ref,m}}{2}$ and both of them move along circular arc IJ .

This modulation strategy is summarized for $0^\circ \leq \theta_m \leq 15^\circ$ in the $\alpha - \beta$ plane, as follows. When tip of $\vec{V}_{ref,m}$ lies within $\triangle ODE$, it is in linear region and solution is given by (19). If tip of $\vec{V}_{ref,m}$ lies within area EDG , it is in overmodulation and solution is given by (20). One of the ways to determine the linear or overmodulation region is to check whether $\theta_m \leq \delta$, where δ is given in (17). Another way is to check whether $\bar{v}_{\alpha m} \leq OE = 1$. In most of the applications, input to the PWM modulator is $(\bar{v}_\alpha, \bar{v}_\beta)$, instead of (M, θ) . Therefore, solutions in (19) and (20) are given in terms of both $(\bar{v}_{\alpha m}, \bar{v}_{\beta m})$ and (M, θ_m) .

$$\bar{V}_{1m} = \bar{V}_{2m} = \frac{\vec{V}_{ref,m}}{2} \triangleq \frac{\bar{v}_{\alpha m}}{2} + j \frac{\bar{v}_{\beta m}}{2} \quad (19)$$

$$\bar{V}_{2m} = \frac{1}{2 \cos \theta_m} e^{j\theta_m} = \frac{1}{2} + j \frac{1}{2} \tan \theta_m = \frac{1}{2} + j \underbrace{\frac{1}{2} \frac{\bar{v}_{\beta m}}{\bar{v}_{\alpha m}}}_{y_{2m}}$$

$$\begin{aligned} \bar{V}_{1m} &= \left(\sqrt{3}M - \frac{1}{2 \cos \theta_m} \right) e^{j\theta_m} = \vec{V}_{ref,m} - \bar{V}_{2m} \\ &= \left(\bar{v}_{\alpha m} - \frac{1}{2} \right) + j(\bar{v}_{\beta m} - y_{2m}) \end{aligned} \quad (20)$$

Above solutions, which are given in terms of $(\bar{v}_{\alpha m}, \bar{v}_{\beta m})$, are also valid for lower-half of sector-1 ($-15^\circ \leq \theta \leq 0^\circ$ in $\alpha - \beta$ plane). Solutions for k^{th} sector, where $k \in \{1, 2, \dots, 12\}$, are derived from (19) and (20), as follows. If $(\bar{v}_\alpha, \bar{v}_\beta)$ are given, k can be identified and subsequently, corresponding modulo-sector voltages can be determined by applying rotational transformation of (21). From this $(\bar{v}_{\alpha m}, \bar{v}_{\beta m})$, \bar{V}_{1m} and \bar{V}_{2m} are determined from (19) or (20) depending upon $\bar{v}_{\alpha m} \leq 1$. If $\bar{v}_{\alpha m} > 1$, \bar{V}_{2m} moves along its boundary (BI in Fig. 7) as boundary of \bar{V}_2 , B_2 , is within the boundary of \bar{V}_1 , B_1 , in modulo sector. From Fig. 5b, one can see that this is true for all odd sectors. In even sectors, B_1 lies within B_2 . Based on this observation, reference voltage vectors per three-phase inverters can be determined from (22), where $x = 1$ and $y = 2$ when k is odd; $x = 2$ and $y = 1$ when k is even. Applying inverse transformation of (7a), modulation signals, \bar{v}_{po} and $\bar{v}_{p'o'}$ where $p \in \{a, b\}$, can be derived. Combining (21), (19), (20) (22) and inverse transformation of (7a), final expressions of these modulating signals are given in (26), (27) and (28) in APPENDIX.

$$\bar{v}_{\alpha m} + j\bar{v}_{\beta m} = (\bar{v}_\alpha + j\bar{v}_\beta) e^{-j(k-1)30^\circ}$$

$$\bar{v}_{\alpha m} = \bar{v}_\alpha \cos(k-1)30^\circ + \bar{v}_\beta \sin(k-1)30^\circ \quad (21a)$$

$$\bar{v}_{\beta m} = -\bar{v}_\alpha \sin(k-1)30^\circ + \bar{v}_\beta \cos(k-1)30^\circ \quad (21b)$$

$$\begin{aligned} (\bar{v}_{\underline{\alpha}} + j\bar{v}_{\underline{\beta}}) &\triangleq \bar{V}_1 = \bar{V}_{xm} e^{j(k-1)30^\circ} \\ (\bar{v}_{\underline{\alpha}'} + j\bar{v}_{\underline{\beta}'}) &\triangleq \bar{V}_2 e^{-j30^\circ} = \bar{V}_{ym} e^{j(k-2)30^\circ} \end{aligned} \quad (22)$$

B. Derivation of harmonic components of the line-neutral voltages

The harmonic components of the line-neutral voltages resulted due to voltage injection in $z_1 - z_2$ plane will be derived in this section. The applied voltages in $\alpha - \beta$ subspace don't contain any harmonics. The voltage injection in $z_1 - z_2$ plane can be determined using (15b) from the knowledge of \bar{V}_1 and \bar{V}_2 ((20) and (19)). These voltage expressions for $0^\circ \leq \theta \leq 360^\circ$ are given in (29) and (30) in APPENDIX, where $k \in \{0, 1, \dots, 5\}$. The harmonic components of $\bar{v}_{z_{1,2}}$ can be obtained using Fourier series analysis. The inverse transformation of (1) enables us to express the line-neutral voltages in the original domains in terms of voltages in transformed domains. Equation (23) expresses a -phase voltage. As $\bar{v}_{o_{1,2}} = 0$ and \bar{v}_α doesn't have any harmonics, therefore the harmonics present in \bar{v}_{z_1} will appear in \bar{v}_{ao} with a scaling factor of $\frac{1}{\sqrt{3}}$. The analytical closed-form expression of n^{th} harmonic components of \bar{v}_{ao} is given in (31) in APPENDIX. The order of harmonics present in the line-neutral voltages are $n = 5, 7, 17, 19, \dots, 12m \pm 5$ and the coefficients of (31) become zero for other odd harmonics. It is difficult to give a generalised expression for $\int_0^\delta \tan \theta \sin(n\theta) d\theta$ in terms of n and δ . Compared to other harmonics present in the line-neutral voltages, impacts of 5^{th} and 7^{th} harmonics are significant due to low impedance of equivalent circuit in $z_1 - z_2$ plane. Therefore, results of this definite integration for $n = 5$ and $n = 7$ are given in (32) and (33) in terms of δ . For a given M , one can obtain the analytical estimation of 5^{th} and 7^{th} harmonics present in the line-neutral voltages by using (17), (31), (32) and (33). As $\bar{v}_\alpha(\theta) = \bar{v}_\beta(\theta + 90^\circ)$ and $\bar{v}_{z_1}(\theta) = \bar{v}_{z_2}(\theta + 90^\circ)$, it is possible to show that the other line-neutral voltages will have odd harmonics spectra similar to \bar{v}_{ao} .

$$\bar{v}_{ao} = \frac{1}{\sqrt{3}}(\bar{v}_\alpha + \bar{v}_{z_1} + \bar{v}_{o_1}) \quad (23)$$

C. Comparisons of modulation techniques based on analysis

Fig. 8 and 9 compare the modulation signal, \bar{v}_{ao} , and \bar{v}_{z_1} , respectively, of the proposed technique and the existing overmodulation techniques of ASIM at $M = 0.597$. In Fig. 8, the proposed strategy has almost similar waveform to $SVOVM4-2$ and both of these strategies have less distortion compared to $SVOVM4-1$ and $CSVPWM$. As $\bar{v}_{z_1}(\theta) = \bar{v}_{z_2}(\theta + 90^\circ)$, waveform of \bar{v}_{z_2} is not shown. Voltage injection by $CSVPWM$ in $z_1 - z_2$ plane is significantly higher compared to other three techniques and it is plotted in different scale in Fig. 9. Among the rest three techniques, \bar{v}_{z_1} of $SVOVM4-2$ and the proposed technique are almost similar and they are better compared to $SVOVM4-1$. \bar{v}_{z_1} of $SVOVM4-2$ is zero for some θ whereas it is non-zero in case of the proposed technique at $M = 0.597$.

TABLE II: Comparison of harmonic performances *CSVPWM* with other three techniques at $M = 0.597$ (in %)

	5 th	7 th	THD	WTHD
<i>CSVPWM</i>	8.8	4.36	16.55	3.13
<i>SVOVM4-1</i>	2.15	0.40	4.03	0.73
<i>SVOVM4-2</i>	1.08	0.91	2.38	0.42
Proposed technique	1.01	1.01	2.39	0.41

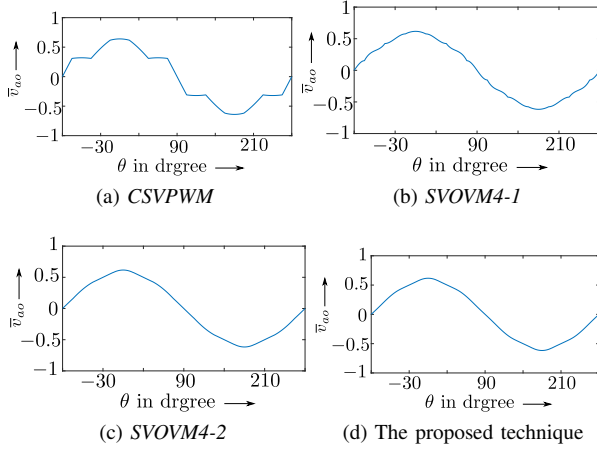


Fig. 8: Comparison of modulating signals at $M = 0.597$

Although the proposed technique or *SVOVM4-1*, *SVOVM4-2* try to minimize THD in line-neutral voltages in order to minimize the injected voltage in $z_1 - z_2$, the main objective is to reduce the current THD in order to reduce the copper loss in $z_1 - z_2$ plane. As leakage inductance becomes predominant over winding resistance for higher order harmonics, therefore current THD reduction is equivalent to reduction of weighted THD (*WTHD*) whose expression is given below.

$$WTHD = \frac{\sqrt{\sum_{n=12m \pm 5} (\frac{\bar{v}_{ao}|_n}{n})^2}}{V_o}$$

The harmonic performance of *CSVPWM* is worse by several orders compared to other three techniques over the entire range of $0.577 \leq M \leq 0.597$. Table II compares 5th harmonic, 7th harmonic, THD and WTHD of line-neutral voltages of all four techniques at $M = 0.597$. Therefore, Fig. 10 compares those three techniques which have comparable harmonic performances. Harmonics up to 100th order have been considered for the computation of WTHD. 5th harmonic performance of the proposed technique is better than *SVOVM4-1* and *SVOVM4-2* whereas *SVOVM4-1* is the best technique in terms of 7th harmonic injection. THD and WTHD performances of the proposed technique and *SVOVM4-2* are almost similar and they are better compared to *SVOVM4-1*. THD performance of *SVOVM4-2* is marginally better than the proposed technique whereas WTHD performance of proposed technique is better compared to *SVOVM4-2* at higher M . As WTHD is the indicator of copper loss, the proposed technique will give the lowest copper loss among all four overmodulation techniques, as we will see in section VII.

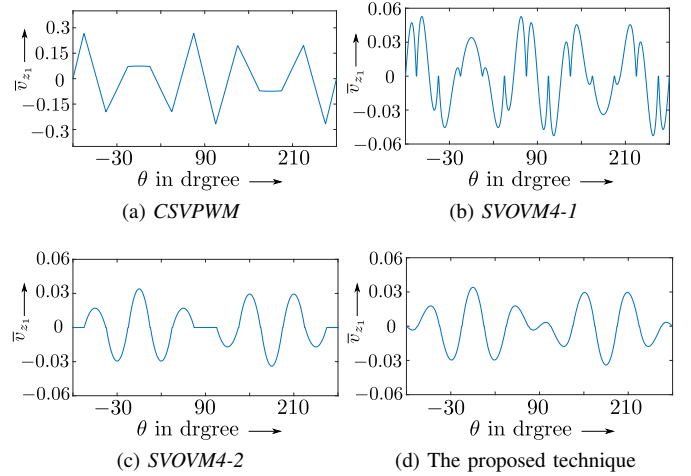


Fig. 9: Comparison of \bar{v}_{z_1} at $M = 0.597$

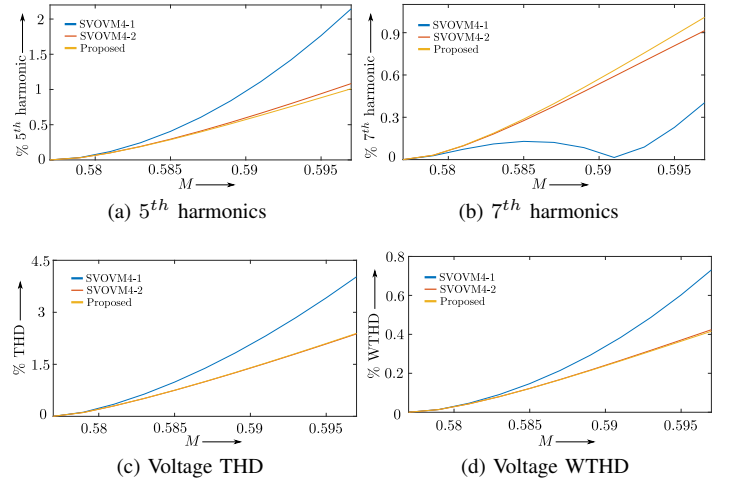


Fig. 10: Comparison of harmonic performances

D. Comparisons of two-level 6 ϕ inverter fed ASIM and three-level NPC fed 3 ϕ -IM

As both multi-phase machine and multi-level converter configurations lead to reduction of voltage stress of the devices which is useful in medium-voltage high-power drives, comparison of these two configurations is important. For a given induction machine of rated power, it can be shown that the blocking voltage rating of the twelve switches in two-level 6 ϕ inverter fed ASIM configuration with proposed technique is lesser than that of three-level neutral-point-clamped (NPC) inverter fed 3 ϕ induction machine (IM) configuration. Current ratings of both the configurations are same. The later needs six additional clamping diodes. In case of fault in one phase, the de-rating factors of ASIM and 3 ϕ -IM are $\frac{1}{6}$ and $\frac{1}{3}$, respectively, which shows that fault-tolerance of two-level 6 ϕ inverter fed ASIM is higher as compared to three-level NPC fed 3 ϕ -IM.

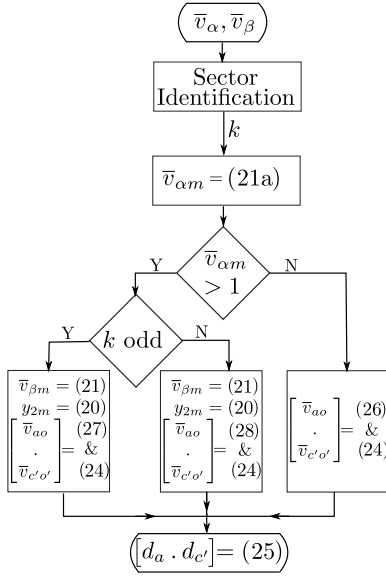


Fig. 11: Flowchart of the proposed strategy

VI. IMPLEMENTATION STRATEGY AND COMPUTATIONAL BURDEN OF THE PROPOSED TECHNIQUE

Following steps are required in order to determine six duty ratios of top switches of six half-bridge legs of Fig. 1 from the given \bar{v}_α and \bar{v}_β pair. By comparing these duty ratios with triangular carrier, PWM modulator block generates six gating pulses of top six switches and six complemented gating pulses with fixed dead-time for bottom switches.

- 1) **Step-1 Sector Identification:** Depending upon signs of \bar{v}_α and \bar{v}_β , the reference voltage vector, $\vec{V}_{ref} \triangleq \bar{v}_\alpha + j\bar{v}_\beta$, may lie in one of the four quadrants in $\alpha - \beta$ plane. Each quadrant consists of four sectors, as shown in Fig. 5b. Maximum three inequalities need to be checked for identifying the exact sector within one quadrant. For example, following inequalities need to be checked sequentially from $l = 0$ to $l = 2$ to identify the sector in first quadrant.

$$\bar{v}_\beta \leq \bar{v}_\alpha \tan((2l + 1)15^\circ); \quad l \in \{0, 1, 2\}$$

2 comparisons (COMPs) are required to determine the quadrant and maximum 3 multiplications (MULs) and 3 COMPs are required to check the three inequalities within a quadrant. Once the sector, k , is identified, remaining calculations are performed in the corresponding sector within if-else block.

- 2) **Step-2 Region Identification:** After identifying k , rotational transformation of (21) is applied to find $\bar{v}_{\alpha m}$, as follows.

$$\bar{v}_{\alpha m} = \bar{v}_\alpha \cos((k - 1)30^\circ) + \bar{v}_\beta \sin((k - 1)30^\circ)$$

If $\bar{v}_{\alpha m} \leq 1$, \vec{V}_{ref} is in linear region. Otherwise, it is in overmodulation region. Maximum 2 MULs, 1 addition (ADD) and 1 COMP are required to identify the region.

- 3) **Step-3 Determination of modulation signals:** Based on linear or overmodulation region of operation and k , 4

modulation signals can be calculated, which are given in (26), (27) and (28) in APPENDIX. 7 MULs and 3 ADDs/subtractions (SUBs) are required to compute 4 modulation signals in linear region. In overmodulation region, $\bar{v}_{\beta m}$ is calculated first from (21). Similar to $\bar{v}_{\alpha m}$ calculation, it requires 2 MULs and 1 ADD. With pre-determined $\bar{v}_{\alpha m}$ and $\bar{v}_{\beta m}$, y_{2m} , which is given in (20), is evaluated by 1 division (DIV) and 1 MUL. The coefficients in (27) or (28) are constant numbers for a given k , therefore, they needn't to be calculated on-line or stored in a table. After y_{2m} is calculated, maximum 4 MULs and 7 ADDs/SUBs are required to compute 4 modulation signals from (27) or (28) at any sector k . Although this isn't obvious from (27) and (28), one can see after plugging any k within $\{1, 2, \dots, 12\}$ that few coefficients of (27) and (28) become either 1 or 0 and only aforementioned computations are required. For two isolated neutral connection, as shown in Fig. 1, \bar{v}_{co} and $\bar{v}_{c'o'}$ can be calculated as (24) and additional 2 ADDs/SUBs are required to do so.

$$\bar{v}_{co} = -(\bar{v}_{ao} + \bar{v}_{bo}); \quad \bar{v}_{c'o'} = -(\bar{v}_{a'o'} + \bar{v}_{b'o'}) \quad (24)$$

- 4) **Step-4 Determination of duty ratios:** This paper uses $\frac{1}{2} + \frac{mid}{2}$ common-mode voltages per three-phase inverters. Therefore, six duty ratios are calculated as (25). Here $x \in \{a, b, c\}$ and $y \in \{a', b', c'\}$, mid and mid' are the middle values of three modulation signals of two three-phase inverters, respectively. Maximum 6 COMPs are required to determine mid and mid' and 8 ADDs, 2 MULs, 2 assign operations are required to calculate all duty ratios.

$$d_x = \bar{v}_{x0} + (1 + mid)\frac{1}{2}; \quad d_y = \bar{v}_{y0'} + (1 + mid')\frac{1}{2} \quad (25)$$

Fig. 11 shows the algorithm to implement the proposed strategy in flowchart format. The computational burden of the above steps are shown in Table III.

When the computational burden of overmodulation techniques of ASIM are compared, it is seen that *CSVPWM* needs lowest computation as it is devoid of calculation of $z_1 - z_2$ plane voltages. But, *CSVPWM* causes substantially higher copper loss, as we will see in section VII, and therefore, it is needless to compare with the proposed technique. As *SVOVM4-1* and *SVOVM4-2* have harmonic performance comparable to the proposed technique, they are compared. It is mentioned in [10] that off-line calculated lookup table based *SVOVM4-1* and *SVOVM4-2* don't have any significant difference in execution time. Therefore, computational burden of *SVOVM4-2* is compared with the proposed technique. The scaling factor of transformation matrix used by [10] is different from (1). For the sake of continuity, all the expressions related to *SVOVM4-2* are re-evaluated with respect to (1) in the following discussion.

The implementation strategy of *SVOVM4-2* is discussed in [10]. 1) Sector identification (step-1), same as discussed before, is required to determine k . 2) The second step, i.e., region identification, consists of two sub-steps-2a) Calculation of $\bar{v}_{\alpha, \beta m}$ from the given $\bar{v}_{\alpha, \beta}$ using (21), which requires

TABLE III: Comparison of Computational Burdens of the Proposed Technique and *SVOVM4-2*

	ADD/SUB		MUL/DIV		COMP	
	Proposed	<i>SVOVM4-2</i>	Proposed	<i>SVOVM4-2</i>	Proposed	<i>SVOVM4-2</i>
Step-1	0	0	3	3	5	5
Step-2	1	3	2	5	1	3
Step-3	10	19	8	17	0	0
Step-4	8	8	2	2	6	6
Total	19	30	15	27	12	14

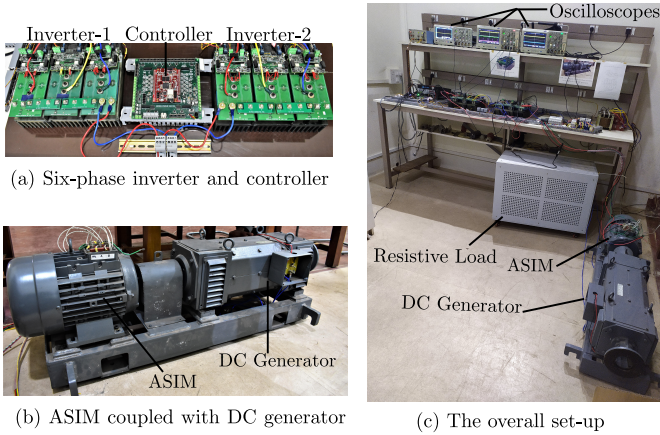


Fig. 12: Experimental set-up

TABLE IV: Steady-state Equivalent Circuit Parameters of ASIM

Per phase stator and rotor resistances	0.675 Ω
Per phase stator and rotor leakage inductances	3.75 mH
Per phase magnetizing inductance	0.168 H

TABLE V: Operating Conditions of Experiments and Simulations

Peak fundamental line-neutral voltage (V_o)	120 $\sqrt{2}$ V
Fundamental frequency (f_o)	50 Hz
Output power	4.2 kW
Switching frequency ($F_s = \frac{1}{T_s}$)	5 kHz

TABLE VI: Simulation based comparison of copper losses in $z_1 - z_2$ plane (in Watt)

M	0.581	0.585	0.589	0.593	0.597
<i>CSVPWM</i>	41.04	41.01	41.06	40.98	40.84
<i>SVOVM4-1</i>	0.37	0.43	0.63	1.34	2.61
<i>SVOVM4-2</i>	0.42	0.47	0.68	0.98	1.42
Proposed technique	0.36	0.47	0.54	0.80	1.09

4 MULs and 2 ADDs; 2b) Zone identification based on $\bar{v}_{\alpha,\beta m}$. If $\bar{v}_{\alpha m} > 1$, it is in overmodulation region. The overmodulation region in sector-1 is divided into three zones. Based on $\bar{v}_{\beta m} \leq 0$ and $2\sqrt{3}\bar{v}_{\alpha m} \pm \bar{v}_{\beta m} > (2 + \sqrt{3})$ inequalities are satisfied or not, zone is identified. It requires 1 MUL, 1 ADD/SUB and 3 COMPs. Therefore, step-2 requires 5 MULs, 3 ADD/SUBs, 3 COMPs. 3) The third step, determination of modulation signals, consists of three sub-steps: 3a) Calculation of $\bar{v}_{z_1 m}, \bar{v}_{z_2 m}$ -In overmodulation, $\bar{v}_{z_1 m} = \bar{v}_{\alpha m} - 1$. In zone-1, $\bar{v}_{z_2 m} = 0$. In zone-2 and zone-3, $\bar{v}_{z_2 m} = \pm 2\sqrt{3}\bar{v}_{\alpha} + \bar{v}_{\beta} \mp (2 + \sqrt{3})$. Therefore, maximum 3 ADD/SUBs, and 1 MULs are required. 3b) Find $\bar{v}_{z_1,2}$ from $\bar{v}_{z_1,2 m}$ using co-ordinate transformation $\bar{v}_{z_1} + j\bar{v}_{z_2} = (\bar{v}_{z_1 m} + j\bar{v}_{z_2 m})e^{j(k-1)150^\circ}$ which requires 4 MULs and 2 ADD/SUBs. 3c) From given $\bar{v}_{\alpha,\beta}$ and already calculated $\bar{v}_{z_1,2}$, six modulation signals are computed by taking inverse transformation of (1). The last two columns of the inverse matrix are not required as $\bar{v}_{o1,2} = 0$. This sub-step requires 12 MULs and 14 ADD/SUBs. Therefore, step-3 requires 19 ADD/SUBs, 17 MULs. 4) Step-4, i.e., calculation of six duty ratios from modulation signals, is same as explained for the proposed algorithm. Computational burdens of these four steps are tabulated in Table III.

Comparing computational burdens in Table III, one can see that the proposed technique can be implemented upto $M = 0.5977$ at much reduced computational cost as compared to *SVOVM4-2* without compromising in performances, as can be seen from Fig. 10.

VII. EXPERIMENTAL AND SIMULATION RESULTS

A. Description of the experimental set-up

A 5 kW, two-pole, 50 Hz squirrel-cage ASIM coupled with 5 kW DC generator (acts as load) has been used to validate the modulation strategy experimentally. The steady-state equivalent circuit parameters of this machine are shown in Table IV. 1200 V, 75 A SEMIKRON IGBT (SKM75GB123D) based half-bridge module has been used to build the six-phase inverter. Zynq-7010 based controller platform is used to generate the PWM signals of the inverter. Zynq-7010 from Xilinx is a system on chip (SoC) which has both processor of dual-core ARM Cortex-A9 architecture (PS) as well as Programmable Logic cells (PL), connected through parallel AXI bus. The clock frequency of the processor can go upto 866 MHz. PL has 28k logic cells, 2.1 Mb block RAM, 80 DSP Slices and 100 I/O pins. In this work, clock frequencies of PS and PL are set to 666 MHz and 100 MHz, respectively. Fig. 12 shows the experimental set-up. The six-phase inverter drives the ASIM which is coupled with a separately excited DC

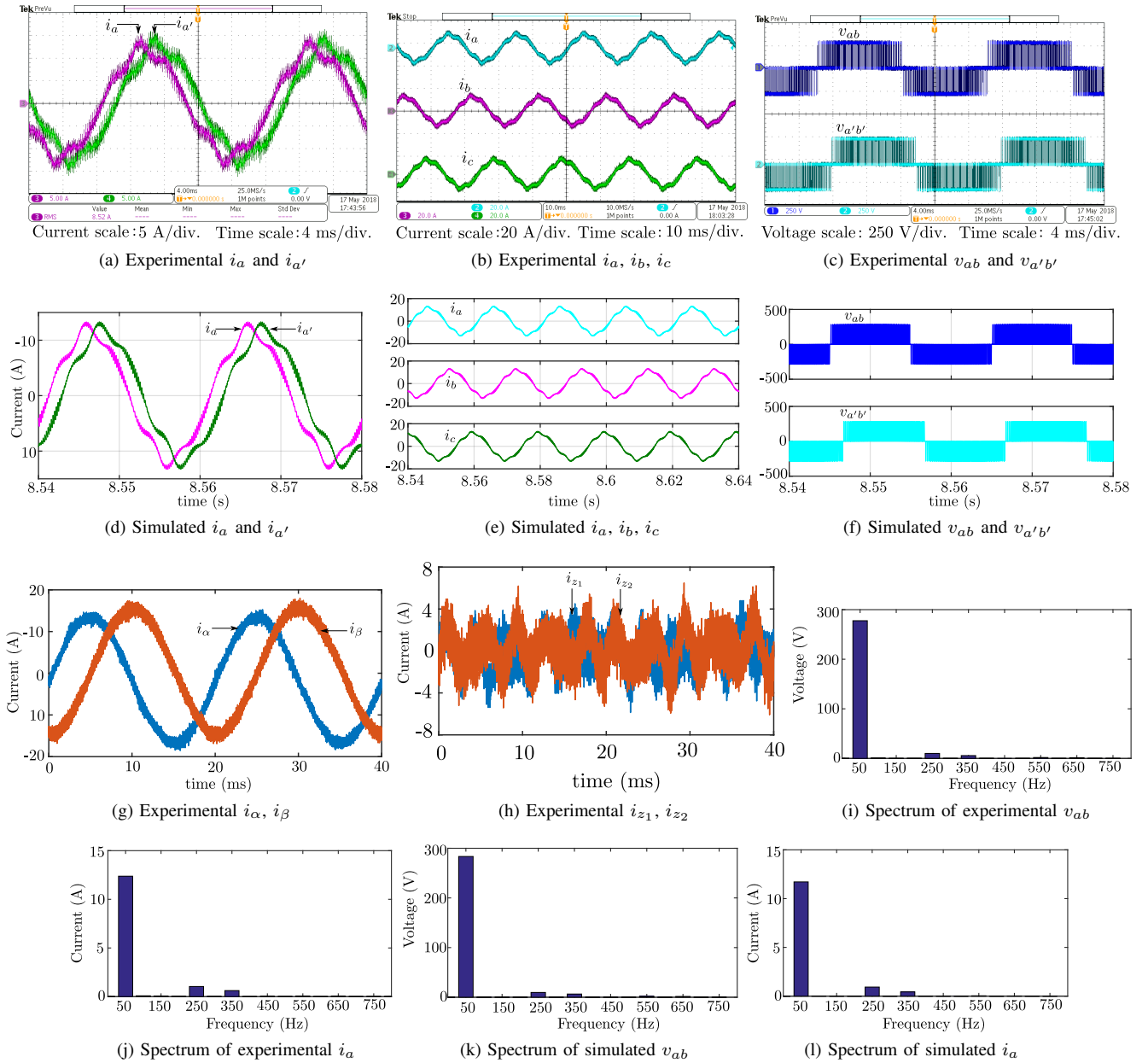
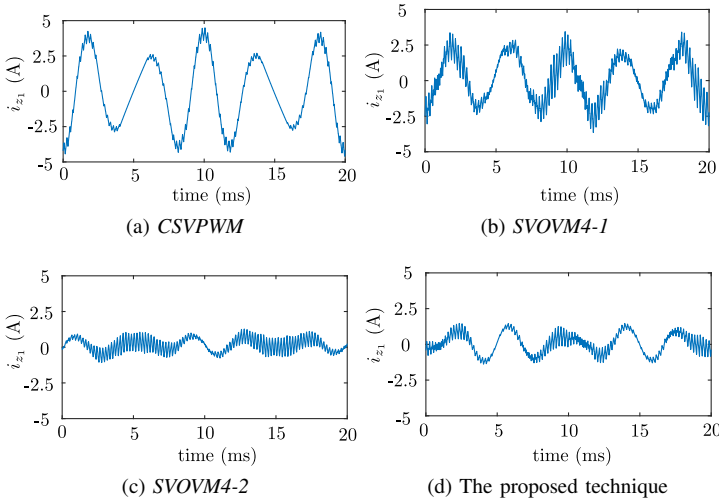
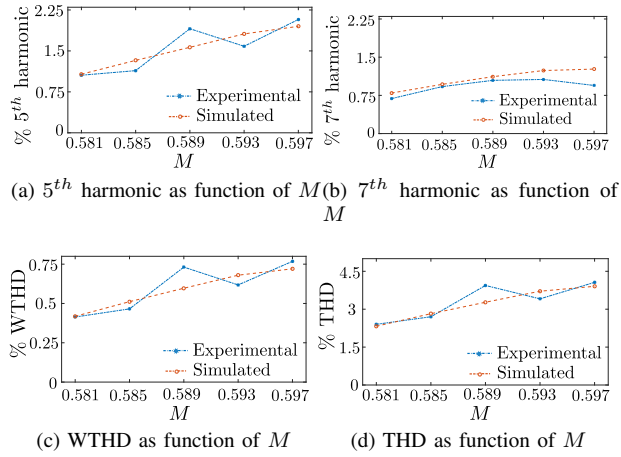


Fig. 13: Experimental and Simulation results of the proposed technique at $M = 0.597$.

generator, which are shown separately in Fig. 12a and 12b. The rated output voltage of generator is 400 V and is connected across a resistive load bank. Tektronix MSO2024B, MDO3104 oscilloscopes, TCP0030A current probes and P5200A differential voltage probes are used during experiment. The overall experimental set-up is shown in Fig. 12c. To validate the proposed strategy in simulation, dynamic model of ASIM is developed in *Matlab Simulink* using the circuit parameters of Table IV. The inverter is simulated using ideal switches. All the experimental and simulation results correspond to the operating conditions shown in Table V. As $MV_{DC} = V_o = 120\sqrt{2} = 169.7$ V, DC-bus voltage V_{DC} has been changed for different M to keep their product constant.

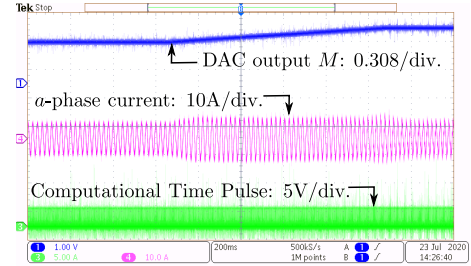
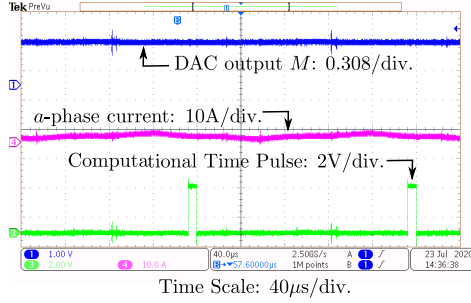
B. Steady-state results

Experiments have been performed at five M values spaced equally within 0.577 – 0.597. Fig. 13 shows the experimental and simulated results at maximum M , i.e., $M = 0.597$. Fig. 13c shows the line-line voltages v_{ab} and $v_{a'b'}$ which are phase shifted by 30° . Simulated line-line voltages are shown in Fig. 13f. Fig. 13a shows the phase currents i_a and $i_{a'}$ (phase shifted by 30° to each other) which closely matches with the simulated waveform shown in Fig. 13d. Phase currents i_a, i_b, i_c are balanced three-phase currents phase-shifted by 120° to each other, as shown in Fig. 13b and 13e. The other set of three-phase currents ($i_{a'}, i_{b'}, i_{c'}$) is similar but phase shifted by 30° with respect to the above set and hence isn't shown. Fig. 13g and 13h show the experimentally obtained currents in $\alpha-\beta$ and z_1-z_2 planes, respectively. To obtain the currents in


 Fig. 14: Simulated i_{z_1} of four techniques at $M = 0.597$

 Fig. 15: Experimental and simulated harmonic performances for $0.577 < M \leq 0.597$

transformed domains, data-points of experimentally obtained phase currents are saved in one excel file and subsequently, transformation T is applied on these data-points in Matlab. Currents in $\alpha - \beta$ plane is sinusoidal, as expected, due to the balanced sinusoidal voltage application in this plane and it validates the proposed strategy. The harmonic spectra of both line-line voltage and phase currents for the experimental and simulated waveforms are shown in Fig. 13i, 13k and 13j, 13l respectively where the presence of 5^{th} and 7^{th} harmonics are prominent, as discussed in section V-B. The peaks of the fundamental component of v_{ab} are 279.9 V and 283.5 V in experimental and simulated results, which means, peak of line-neutral voltages are 161.6 V and 163.7 V respectively. The peaks of fundamental component of i_a in experimental and simulated waveforms are 12.4 A and 11.7 A respectively.

Table VI compares the simulation results of copper losses incurred in $z_1 - z_2$ plane for all four techniques at aforementioned five M using the machine parameters mentioned in table IV. The proposed technique gives the lowest copper loss among these four techniques at higher M which is close


 (a) Open-loop V/f control: $M = 0.448$, $f_o = 45Hz$ to $M = 0.597$, $f_o = 50Hz$


(b) Computational time of the proposed strategy

Fig. 16: Dynamic performance of ASIM and computation time of proposed strategy

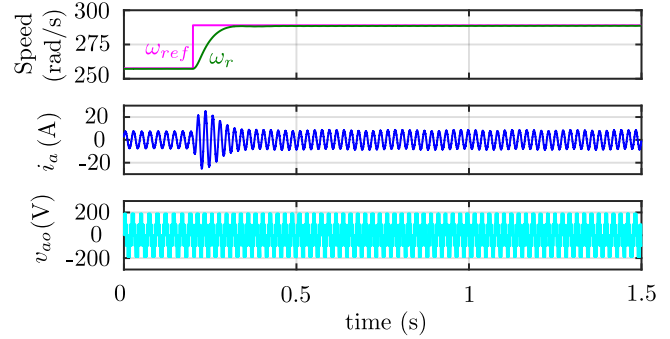
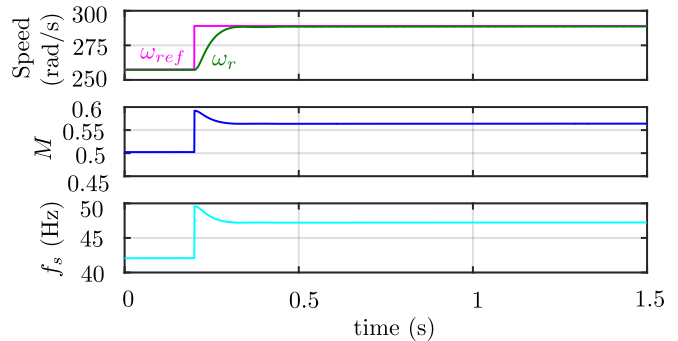

 (a) ω_{ref} , ω_r , i_a and v_{ao} .

 (b) ω_{ref} , ω_r , M and f_s

Fig. 17: Simulated scalar-controlled closed-loop response of ASIM

to the copper loss of *SVOVM4-2*. As this copper loss is related to current waveforms in $z_1 - z_2$ plane (copper loss is equal to $(i_{z_1,RMS}^2 + i_{z_2,RMS}^2) \times r_s$, where $i_{z_1,RMS}$ and $i_{z_2,RMS}$ are RMS values of i_{z_1} and i_{z_2} , respectively, r_s is the resistance seen by the $z_1 - z_2$ plane), i_{z_1} plot for all four techniques at $M = 0.597$ are shown in Fig. 14 which validate Table VI. The waveform of i_{z_2} is similar to i_{z_1} but phase-shifted by 90° , therefore, i_{z_2} plot is not shown here. Fig. 15 compares the performance of the proposed technique, as obtained through experiments and simulations, over the range $0.577 < M \leq 0.597$ at five distinct M values. Close agreements of THD, WTHD, 5th and 7th harmonic performances between experimental and simulated waveforms validates the proposed strategy. The difference between analytical and experimental WTHD, THD, 5th and 7th harmonic estimation arises due to the clamping of the duty ratios of the inverter switches at 0.975 (upper bound) and 0.025 (lower bound) for proper dead-time operation between top and bottom switches of a leg. 1.2 μs dead-time is used between top and bottom switches of the inverter and if the switch ‘on’ time is less than 5 μs , the switch is kept in ‘off’ state. When the duty ratios have not been clamped in simulation (it is possible as ideal switches are used in simulation), these performance indices are closely matching with the analytical results.

C. Results with dynamics

Fig. 16a shows the dynamic performance of ASIM, driven by V/f control, where M (brought out from the controller through DAC), one of the line currents and computational time pulse are captured in one scope. Computational time pulse is a signal that is high when the calculation of PWM modulator is in progress. During this experiment, M and output frequency, f_o , are changed from (0.448, 45 Hz) to (0.597, 50 Hz) in 1 sec. It can be seen that the line current was going through a transient during the ramped increase in M , as expected, and settled back to its original state once the dynamics is over. Fig. 16b shows the zoomed version of Fig. 16a over few sampling cycles to show the computational time pulse. As expected, its frequency is same as the sampling frequency ($F_s = \frac{1}{T_s} = 5$ kHz), and computation time is 8 μs which is much smaller than $T_s = 200 \mu s$.

D. Closed-loop results

The closed-loop performances of scalar-controlled (V/f) ASIM has been simulated in Matlab and shown in Fig. 17. The plant model is similar to 3ϕ induction machine where the steady-state torque is proportional to the slip speed. Proportional-Integral (PI) controller is designed such that the

pole of the mechanical load is cancelled by the zero of the controller and the rise-time of the resultant first order system is 0.15 s. This choice of rise-time results into transient current to be 150% of full load current, which is in the permissible range, [32]. Fig. 17a shows the closed-loop speed, voltage and current waveforms when there is step change of reference speed from 258 rad/s to 289 rad/s. The change in synchronous frequency, f_s , and M are shown in Fig. 17b. (M , f_s) pair are changed from (0.5, 41.87 Hz), i.e. linear region, reach the peak of (0.592, 49.58 Hz), i.e. overmodulation region, and finally settled down to (0.564, 47.24 Hz).

VIII. CONCLUSIONS

This paper derives an important relationship between individual space-vectors of two three-phase inverters and the space-vectors in $\alpha - \beta$ and $z_1 - z_2$ subspaces of six-phase inverter. Using the above relation, an overmodulation (OVM) technique is proposed which modulates two three-phase inverters in order to synthesize the given reference voltage vector in $\alpha - \beta$ subspace. According to proposed strategy, the reference vectors of these two inverters are always phase shifted by 30° . As usual, in linear region, their magnitudes are same and equal to half of the magnitude of reference voltage vector in $\alpha - \beta$ subspace. But in OVM region, their magnitudes are different. The maximum modulation index achievable by this technique is derived analytically and found to be 0.5977. This technique doesn't inject any harmonics in $\alpha - \beta$ subspace and therefore no torque pulsation is created in ASIM. The proposed technique covers 44.44 % of the entire OVM range. Analytical closed form expression of harmonic components of the line-neutral voltages is derived. The proposed strategy injects $n = 5, 7, 17, 19, \dots, 12m \pm 5$ order harmonics in line-neutral voltages. In OVM range, both THD and WTHD of the line-neutral voltages are monotonically increasing functions of modulation index and are 2.4 % and 0.42 % respectively at maximum modulation index. Detailed comparisons of harmonic performances of the proposed technique with other existing techniques reveal that THD, WTHD performances of the proposed technique is better than existing *CSVPWM* and *SVOVM4-1* techniques and almost similar to optimal *SVOVM4-2* whereas comparison of computational burdens of the proposed technique with *SVOVM4-2* shows that the proposed technique can be implemented at much reduced computational cost without involving complex six-dimensional transformation. The proposed technique is validated through experiments and simulation on 4.2 kW hardware prototype.

IX. APPENDIX

In linear region of operation ($\bar{v}_{\alpha m} \leq 1$), (26) are the expressions for modulation signals. Equations (27) and (28) give the modulation signals of odd and even sectors, respectively, in overmodulation operation.

$$\bar{v}_{ao} = \frac{1}{\sqrt{3}}\bar{v}_\alpha; \quad \bar{v}_{a'o'} = \frac{1}{2}\bar{v}_\alpha + \frac{1}{2\sqrt{3}}\bar{v}_\beta; \quad \bar{v}_{bo} = -\frac{1}{2\sqrt{3}}\bar{v}_\alpha + \frac{1}{2}\bar{v}_\beta; \quad \bar{v}_{b'o'} = -\frac{1}{2}\bar{v}_\alpha + \frac{1}{2\sqrt{3}}\bar{v}_\beta; \quad (26)$$

$$\begin{aligned}\bar{v}_{ao} &= \frac{2}{\sqrt{3}}\bar{v}_\alpha + \left(\frac{2}{\sqrt{3}}\sin((k-1)30^\circ)\right)y_{2m} - \frac{1}{\sqrt{3}}\cos((k-1)30^\circ); & \bar{v}_{a'o'} &= \left(-\frac{2}{\sqrt{3}}\sin((k-2)30^\circ)\right)y_{2m} + \frac{1}{\sqrt{3}}\cos((k-2)30^\circ); \\ \bar{v}_{bo} &= -\frac{1}{\sqrt{3}}\bar{v}_\alpha + \bar{v}_\beta - \left(\frac{2}{\sqrt{3}}\sin((k+1)30^\circ)\right)y_{2m} + \frac{1}{\sqrt{3}}\cos((k+1)30^\circ); & \bar{v}_{b'o'} &= \left(\frac{2}{\sqrt{3}}\sin(k30^\circ)\right)y_{2m} - \frac{1}{\sqrt{3}}\cos(k30^\circ);\end{aligned}\quad (27)$$

$$\begin{aligned}\bar{v}_{ao} &= \left(-\frac{2}{\sqrt{3}}\sin((k-1)30^\circ)\right)y_{2m} + \frac{1}{\sqrt{3}}\cos((k-1)30^\circ); & \bar{v}_{a'o'} &= \bar{v}_\alpha + \frac{1}{\sqrt{3}}\bar{v}_\beta + \left(\frac{2}{\sqrt{3}}\sin((k-2)30^\circ)\right)y_{2m} - \frac{1}{\sqrt{3}}\cos((k-2)30^\circ); \\ \bar{v}_{bo} &= \left(\frac{2}{\sqrt{3}}\sin((k+1)30^\circ)\right)y_{2m} - \frac{1}{\sqrt{3}}\cos((k+1)30^\circ); & \bar{v}_{b'o'} &= -\bar{v}_\alpha + \frac{1}{\sqrt{3}}\bar{v}_\beta - \left(\frac{2}{\sqrt{3}}\sin(k30^\circ)\right)y_{2m} + \frac{1}{\sqrt{3}}\cos(k30^\circ);\end{aligned}\quad (28)$$

$$\bar{v}_{z1} = \begin{cases} \left(\frac{1}{\cos\delta} - \frac{1}{\cos\left(\frac{k\pi}{3} - \theta\right)}\right)\cos\theta; & \left(\frac{k\pi}{3} - \delta\right) \leq \theta \leq \left(\frac{k\pi}{3} + \delta\right) \\ \left(\frac{1}{\cos\left(\frac{(2k+1)\pi}{6} - \theta\right)} - \frac{1}{\cos\delta}\right)\cos\theta; & \left(\frac{(2k+1)\pi}{6} - \delta\right) \leq \theta \leq \left(\frac{(2k+1)\pi}{6} + \delta\right) \\ 0; & \text{elsewhere} \end{cases}\quad (29)$$

$$\bar{v}_{z2} = \begin{cases} \left(\frac{1}{\cos\delta} - \frac{1}{\cos\left(\frac{k\pi}{3} - \theta\right)}\right) - \sin\theta; & \left(\frac{k\pi}{3} - \delta\right) \leq \theta \leq \left(\frac{k\pi}{3} + \delta\right) \\ \left(\frac{1}{\cos\left(\frac{(2k+1)\pi}{6} - \theta\right)} - \frac{1}{\cos\delta}\right) - \sin\theta; & \left(\frac{(2k+1)\pi}{6} - \delta\right) \leq \theta \leq \left(\frac{(2k+1)\pi}{6} + \delta\right) \\ 0; & \text{elsewhere} \end{cases}\quad (30)$$

$$\bar{v}_{ao}|_n = \frac{4}{\sqrt{3}\pi} \left[\frac{1}{\cos\delta} \left(1 - \sqrt{3}\cos\left(\frac{n\pi}{6}\right) + \cos\left(\frac{n\pi}{3}\right)\right) \frac{n\cos\delta\sin(n\delta) - \sin\delta\cos(n\delta)}{n^2 - 1} + \frac{1}{\cos\delta} \left(-\sin\left(\frac{n\pi}{2}\right) + \sqrt{3}\sin\left(\frac{n\pi}{3}\right) - \sin\left(\frac{n\pi}{6}\right)\right) \frac{\cos\delta\sin(n\delta) - n\sin\delta\cos(n\delta)}{n^2 - 1} + \left(-1 + \sqrt{3}\cos\left(\frac{n\pi}{6}\right) - \cos\left(\frac{n\pi}{3}\right)\right) \frac{\sin(n\delta)}{n} + \left(\sin\left(\frac{n\pi}{2}\right) - \sqrt{3}\sin\left(\frac{n\pi}{3}\right) + \sin\left(\frac{n\pi}{6}\right)\right) \int_0^\delta \tan\theta\sin(n\theta)d\theta \right]\quad (31)$$

$$\int_0^\delta \tan\theta\sin(5\theta)d\theta = \left[-2\sin\delta + \frac{2}{3}\sin(3\delta) - \frac{1}{5}\sin(5\delta) + \ln\left(\tan\left(\frac{\pi}{4} + \frac{\delta}{2}\right)\right)\right]\quad (32)$$

$$\int_0^\delta \tan\theta\sin(7\theta)d\theta = \left[2\sin\delta - \frac{2}{3}\sin(3\delta) + \frac{2}{5}\sin(5\delta) - \frac{1}{7}\sin(7\delta) + \ln\left(\tan\left(\frac{\pi}{4} - \frac{\delta}{2}\right)\right)\right]\quad (33)$$

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