Overmodulation Techniques of Asymmetrical Six-phase Machine With Optimum Harmonic Voltage Injection

Sayan Paul, Kaushik Basu, Senior Member, IEEE

Abstract—Overmodulation techniques of asymmetrical six-phase machine achieve higher DC-bus utilization by applying voltage in non-energy transfer plane. This results into unwanted current and associated copper loss. Existing overmodulation technique minimizes this voltage from space-vector perspective with pre-defined set of four active vectors. To find the best technique, one needs to perform the above minimization problem with all possible sets of active vectors with which higher voltage gain can be attained. So, this requires evaluation of large number of cases. This paper formulates the above minimization problem in terms of average voltage vectors of two three-phase inverters, where active vectors need not to be specified beforehand and thus the analysis is more general. Sixteen possible techniques with different set of active vectors are derived following the above analysis, which attain minimum voltage injection in non-energy transfer plane. This paper also identifies one of these sixteen techniques, which can be implemented without involving complex six-dimensional transformation. The above technique is validated through experiments on six-phase induction motor and simulations in Matlab at 3.5 kW power level.

Index Terms—Multiphase machines, pulse width modulation (PWM), overmodulation, asymmetrical six-phase induction motor, six-phase drives, harmonic minimization.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( x )</td>
<td>Switching cycle average of variable ( x )</td>
</tr>
<tr>
<td>( \vec{V}_2 + j\vec{V}_3 )</td>
<td>3φ average voltage space-vector of Inverter-1.</td>
</tr>
<tr>
<td>( \vec{V}_2' + j\vec{V}_3' )</td>
<td>3φ average voltage space-vector of Inverter-2.</td>
</tr>
<tr>
<td>( \vec{V}_z + j\vec{V}_z' )</td>
<td>6φ average voltage space-vector in ( \alpha - \beta ) plane.</td>
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<tr>
<td>( \vec{V}<em>{z1} + j\vec{V}</em>{z2} )</td>
<td>6φ average voltage space-vector in ( z_1 - z_2 ).</td>
</tr>
<tr>
<td>( \vec{V}_{ref} )</td>
<td>Reference voltage vector in ( \alpha - \beta )</td>
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<td>( M )</td>
<td>Modulation index</td>
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I. INTRODUCTION

ASYMMETRICAL six-phase machine (ASPM), one of the most popular multi-phase machines, has two sets of balanced three-phase (3φ) windings which are spatially shifted by 30° electrical, as shown in Fig. 1. This machine is attractive in high power applications compared to 3φ due to better fault tolerance for having more number of phases, reduced rating of per-phase power-electronic drive unit, less susceptibility towards excitation harmonics, [1], [2].

To model ASPM with sinusoidal winding distribution, [3] proposed a 6 × 6 orthonormal transformation matrix, \( T \). \( T \) transforms the six-dimensional quantities from original frame to three two-dimensional orthogonal subspaces, as \( \alpha - \beta \), \( z_1 - z_2 \) and \( o_1 - o_2 \), and the dynamic models of ASPM are decoupled in these subspaces. \( \alpha - \beta \) subspace is associated with electromechanical energy transfer and therefore, this subspace should be excited with balanced fundamental voltage in order to generate ripple-free torque in ASPM. The air-gap flux doesn’t appear in other subspaces and hence torque remains unaffected with the applied voltages in these subspaces. With the balanced winding with two isolated neutrals, as shown in Fig. 1, \( o_1 - o_2 \) plane can’t be excited. Although applied voltage in \( z_1 - z_2 \) doesn’t create torque pulsation, it causes large circulating current and associated copper loss due to low impedance of the equivalent circuit. Therefore, applied voltage in \( z_1 - z_2 \) should be as minimum as possible.

Linear modulation techniques of six-phase (6φ) inverter fed ASPM don’t apply any average voltage in \( z_1 - z_2 \) subspace. These techniques can be broadly classified into two categories: space-vector based techniques (SVPWM) and two-inverter (TINV) based techniques. In SVPWM techniques, desired voltages in \( \alpha - \beta \) and zero voltages in \( z_1 - z_2 \) are synthesised by the application of at least four active vectors and zero vectors. SVPWM based linear modulation techniques with different choice and number of active vectors and their performance comparisons are presented in [3]–[10]. To avoid complex implementation strategy of SVPWM based techniques, [11], [12] modulate two 3φ inverters with reference voltage vectors of equal magnitude at a phase difference of 30°. The resulting

Fig. 1: Six-phase inverter fed ASPM
techniques don’t apply any voltage in $z_{1} - z_{2}$ subspace. These are labelled as TINV techniques in this paper. If modulation index ($M$) is defined as the ratio of peak of fundamental line-neutral voltage and DC-bus voltage, the maximum $M$ attainable by both of these categories is 0.577.

Overmodulation (OVM) techniques apply a group of harmonics in line-neutral voltages to attain $0.577 < M \leq 0.622$ so that these harmonics appear only in $z_{1} - z_{2}$ without affecting fundamental operation in $\alpha - \beta$. [13] proposed one such technique, which is known as CSVPWM ([3]), where desired voltage vector in $\alpha - \beta$ was synthesised using two adjacent active vectors. This resulted into large circulating current in $z_{1} - z_{2}$ plane, [3]. To minimize this, [6] had proposed two space-vector based OVM (SVOV M) techniques, named as $SVOV M4-1$ and $SVOV M4-2$ in this paper, which use two pairs of active vectors adjacent to the reference voltage vector in $\alpha - \beta$ without any zero-vector. Each pair consists of largest and medium collinear vectors of $\alpha - \beta$. With these vectors, $SVOV M4-2$ formulated an optimization problem to achieve minimum RMS voltage in $z_{1} - z_{2}$ and it is the best known technique among all existing OVM techniques of ASPM. It should be mentioned here that OVM of symmetrical five-phase machine with one isolated neutral also requires four-dimensional modulation like ASPM, [14]–[18]. [14], [15] proposed OVM technique of five-phase machine using similar pairs of voltage vectors as [6]. [19] proposed TINV based OVM techniques in which two $3\phi$ inverters were operated through OVM schemes of $3\phi$ inverters, as proposed by [20], in $30^\circ$ phase difference. But, these techniques resulted into injection of harmonics in $\alpha - \beta$.

In SVPWM based voltage minimization technique, like $SVOV M4-2$ of [6], the active vectors need to be decided beforehand. As there are multiple ways of choosing these active vectors, one needs to perform the minimization problem for all such choices to find the best technique. In this context, this paper makes following three contributions.

- The average voltage space-vectors of $6\phi$ inverter in $\alpha - \beta$ and $z_{1} - z_{2}$ planes are expressed in terms of average voltage space-vectors of two $3\phi$ inverters. Then, the aforementioned voltage minimization in $z_{1} - z_{2}$ is formulated in terms of these $3\phi$ average voltage vectors. Thus, the problem formulation doesn’t require specification of the active vectors beforehand and the analysis is more general.
- Based on the above analysis, the whole OVM region is divided into two parts. Optimal solution can be realized by eight possible sets of switching states in one part and two possible sets in another part. Combining these two, sixteen possible techniques are obtained to attain optimal solution. $SVOV M4-2$ is one such technique.
- Finally, this paper presents a two-inverter based optimal OVM technique which can be implemented without involving complex $6\phi$ transformation and only requires knowledge of $3\phi$ inverter modulation.

The organization of the paper is as follows. Section II briefly describe the modeling of $6\phi$ machine and converter and existing space-vector based OVM techniques. Section III formulates the optimization problem to attain global minimum voltage injection in $z_{1} - z_{2}$ plane and section IV derives few SVOVM techniques to attain this global minima. Finally, carrier based implementation of one of the SVOVM techniques with optimum harmonic voltage injection is discussed in section-V and validated through experiments and simulations in section VI. The paper has been concluded in section VII.

II. EXISTING OVERMODULATION TECHNIQUES OF ASYMMETRICAL SIX-PHASE MACHINE

Fig. 1 shows the connection diagram of six stator phases of asymmetrical six-phase machine (ASPM) with two-level $6\phi$ inverter. The $6\phi$ inverter can be viewed as two $3\phi$ inverters, inverter-1 and inverter-2. The terminals of the six stator phase windings are $a, b, c, a', b', c'$, which are directly connected to the poles of the six legs of the inverter. These two sets of balanced $3\phi$ windings are connected in star fashion with two isolated neutral points, $o$ and $o'$, and these two sets are spatially shifted by $30^\circ$.

A. Machine Modeling

$$X_i = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1/2 & -1/2 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\ 1 & -1/2 & -1/2 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} X_j^T$$

$$X_i = \begin{bmatrix} x_a & x_b & x_c & x_{a'} & x_{b'} & x_{c'} \end{bmatrix}^T$$

$$X_j = \begin{bmatrix} x_a & x_b & x_c & x_{a'} & x_{b'} & x_{c'} \end{bmatrix}^T$$

[3] has proposed a $6 \times 6$ orthonormal transformation matrix $T$, as given in (1), to analyse the dynamic model of ASPM in transformed domain, $\alpha - \beta - z_{1} - z_{2} - o_{1} - o_{2}$. The vector quantities in (1) can be voltage, current or flux-linkage etc. Followings are few important properties of ASPM in transformed domain.

1) The transformed six dimensions are orthogonal to each other and can be grouped as three two-dimensional subspaces, viz., $\alpha - \beta$, $z_{1} - z_{2}$ and $o_{1} - o_{2}$, respectively. Dynamic equivalent circuits of ASPM in these subspaces are decoupled to each other.

2) The coupling of flux-linkages and currents between stator and rotor is seen only in $\alpha - \beta$ subspace. Both the dynamical and steady-state equivalent circuits in this subspace are similar to the equivalent circuits of $3\phi$ machine with $3\phi$ Clarke’s transformation. This is the only subspace associated with electromechanical energy transfer and hence $\alpha - \beta$ plane should be excited with balanced fundamental voltage so as to rotate the ASPM without any torque ripple.

3) The dynamic and steady-state equivalent circuits in $z_{1} - z_{2}$ and $o_{1} - o_{2}$ subspaces consist of winding resistance and leakage inductance. These subspaces are not associated with electromechanical energy transfer. Although these subspaces don’t create any torque pulsation, injection of
TABLE I: Switching States of Three-phase Inverter

<table>
<thead>
<tr>
<th>Switching State</th>
<th>Label</th>
<th>Switching State</th>
<th>Label</th>
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<tbody>
<tr>
<td>000</td>
<td>0</td>
<td>001</td>
<td>4</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>011</td>
<td>5</td>
</tr>
<tr>
<td>110</td>
<td>2</td>
<td>101</td>
<td>6</td>
</tr>
<tr>
<td>010</td>
<td>3</td>
<td>111</td>
<td>7</td>
</tr>
</tbody>
</table>

vectors in these subspaces should be low to reduce the RMS current and associated loss in these subspaces. For two isolated neutrals (as shown in Fig. 1), \( i_{a1}, i_{o2} \) in (1), corresponding to line currents, are zero. So, \( v_{o1}, v_{o2} \), corresponding to line-neutral voltages, for balanced six-phase windings will be always zero for any of the switching states of the inverter. Therefore, discussion of modulation in \( a_1 - o_2 \) subspace is omitted in subsequent sections.

B. Converter Modeling

As each of the two-level \( 3\phi \) inverters has 8 permissible switching states, \( 6\phi \) inverter of Fig. 1 has \( 8 \times 8 = 64 \) switching states. These states are labelled by using two numbers, like \((x, y')\), where \(x, y'\) denote the switching states of inverter-1 and inverter-2, respectively, and \(x, y \in \{ 0, 1, \ldots, 7 \}\). Table I lists 8 switching states of \( 3\phi \) inverter. Each entry under ‘Switching State’ column has three binary digits which denote the switching states of top switches of \( a, b, c \) legs, respectively, of a \( 3\phi \) inverter. Here, 0 represents ‘off’ and 1 represents ‘on’. The bottom switch of any leg is complementary switched with respect to top switch. Therefore, 101 state implies top switches of \( a \) and \( c \) legs and bottom switch of \( b \) legs are in ‘on’ state. So, switching state \((1, 6')\) of \( 6\phi \) inverter means top switches of \( a, a', c' \) and bottom switches of \( b, c, b' \) are ‘on’; other switches are ‘off’.

The line-neutral voltages generated by each of the switching states can be mapped into transformed domain by applying \( T \) of (1). For example, line-neutral voltage vector in original domain, \( V_j \) in (1), generated by state \((1, 6')\) is given as, \( V_j(1,6') = \frac{V_{DC}}{\sqrt{3}} [ 2 -1 -1 1 -2 1 ] \). The corresponding mapping in transformed domain, \( \tilde{V}_j \), is obtained by applying \( T \) on \( V_j \). Combining \( \alpha - \beta \) and \( z_1 - z_2 \) voltages together, we get, \((v_{\alpha} + jv_{\beta})(1,6') = \frac{2V_{DC} \cos 15^\circ}{\sqrt{3}} e^{-j15^\circ} \) and \((v_{z_1} + jv_{z_2})(1,6') = \frac{2V_{DC} \cos 75^\circ}{\sqrt{3}} e^{-j75^\circ} \). Similarly, 64 states can be mapped in transformed domain. It gives 60 active vectors (48 distinct vectors) and 4 zero vectors in both \( \alpha - \beta \) and \( z_1 - z_2 \). The mapping of these 64 states can be referred from [3]. The tips of 12 largest active vectors in \( \alpha - \beta \) form a do-decagon. \( \alpha - \beta \) plane within this do-decagon can be divided in 12 equivalent sectors. Fig. 2 shows the mapping of those 16 states in \( \alpha - \beta \) and \( z_1 - z_2 \) which are required to explain the space-vector based overmodulation (OVM) techniques with reference voltage vector, \( \tilde{V}_{ref} \), within sector-1 of \( \alpha - \beta \) plane. \( \tilde{V}_{ref} \) is defined as \( \tilde{V}_{ref} = \overline{a_1} + j\overline{b_1} \); bar represents the average value over a switching cycle. Among these 16 states, 4 are zero states and they are denoted by \((z, z')\), where \(z \in \{ 0, 7 \}\).

Remaining 12 states give 9 distinct active vectors, shown in Fig. 2, as two zero states of one \( 3\phi \) inverter in combination with one active state of other \( 3\phi \) inverter give rise to same vector in both \( \alpha - \beta \) and \( z_1 - z_2 \) planes. Therefore, each of the active vectors among \((1, 1'), (1, z') \) and \((z, 6')\) in Fig. 2 is the resultant of two states. The boundary of the do-decagon is shown by the dotted line in Fig. 2a and \( 12^{th}, 1^{st} \) and \( 2^{nd} \) sectors are indicated in the figure.

In this paper, modulation index \((M)\) is defined as the ratio of peak of the fundamental line-neutral voltage and DC-bus voltage. OVM techniques are referred to those techniques which don’t apply any harmonic voltage in \( \alpha - \beta \) but do apply harmonic voltages in \( z_1 - z_2 \) plane in order to achieve higher \( M \). It is well-known in ASYM literature that linear techniques attain \( M \) up to 0.577 whereas OVM techniques can achieve \( M \) up to 0.622. What follows is the brief discussion on existing OVM techniques of ASYM and their limitations.

C. Existing SVOVM techniques of ASPM and its limitations

Suppose, we want to synthesize \( \tilde{V}_{ref} \) in sector-1 of \( \alpha - \beta \) plane, as shown in Fig. 2. Existing two-vector based SVOVM technique of [13], later named as CSVPWM in [3], uses two large active vectors adjacent to \( \tilde{V}_{ref} \), like \((1,6'), (1,1')\) and zero-vector. Harmonic voltage injected by CSVPWM in \( z_1 - z_2 \) plane is significant, [3]. [6] has proposed two OVM techniques, which are termed as SVOVM4-1 (space-vector based OVM technique using 4 active vectors) and SVOVM4-2 in this paper, in order to reduce harmonic voltage injection in \( z_1 - z_2 \) plane. It uses 2 large and 2 medium vectors adjacent to \( \tilde{V}_{ref} \), like \((6,1'), (1,6'), (2,6')\) and \((1,1')\) in sector-1. With this four vectors, SVOVM4-2 minimizes average voltage injected in \( z_1 - z_2 \) plane, \( \overline{\Delta V_{z1}^2} + \overline{\Delta V_{z2}^2} \). Similar OVM techniques for five-phase drives were proposed by [14]–[16]. Suppose, the minimal voltage injection in \( z_1 - z_2 \) plane obtained by SVOVM4-2 is denoted as \( |\Delta V_{z1} + j\Delta V_{z2}|_2L+2M \).

Let us consider an alternative 4 vector strategy, that applies 2 large, 1 medium and 1 small vectors adjacent to \( \tilde{V}_{ref} \) in sector-1 of \( \alpha - \beta \) plane: \((1,z'), (1,1'), (1,6') \) and \((2,6')\). For this particular choice of vectors, we can find the optimal modulation strategy following the similar analysis as outlined in [6] and suppose the optimal voltage is denoted by \( |\Delta V_{z1} + j\Delta V_{z2}|_2L+1M+1S \). We observed that \( |\Delta V_{z1} + j\Delta V_{z2}|_2L+1M+1S \geq |\Delta V_{z1} + j\Delta V_{z2}|_2L+2M \) for all possible \( \tilde{V}_{ref} \) in overmodulation region of upper-half of sector-I. But, if \((6,1')\) is used instead of \((1,1')\) (where both \((1,1')\) and \((1, z')\) are small vectors adjacent to \( \tilde{V}_{ref} \) in sector-1) keeping other three vectors

![Fig. 2: Mapping of 16 neighbouring switching states of sector-1 results into 10 distinct vectors](image-url)
same, the optimal solution obtained by that set is same as $|\vec{v}_1 + j\vec{v}_{2L+2M}|$.

The above example clearly establishes following facts: 1) There are more than one set of vectors that can be used to synthesize a reference voltage vector in $\alpha - \beta$ plane in overmodulation. 2) Following the procedure outlined in [6], one can find the duty ratios of this specific set of vectors that will minimize the length of the average applied vector in $z_1 - z_2$ plane. 3) This minimum length depends on the choice of the set of vectors. So, for a given reference voltage vector in overmodulation region, unless we compute the minimum lengths of the average voltage vectors in $z_1 - z_2$ plane by every possible techniques, we do not know a) SVOVM-4-2 in [6] is the technique that attains smallest among all minimum lengths of the average voltage vectors in $z_1 - z_2$ plane (we can call it global minimum), b) even if that is true, are there techniques other than SVOVM-4-2 in [6] that also achieves the same global minimum. In order to find the best strategy, one needs to do the following: a) Determine all possible sets of adjacent vectors for a given $\vec{v}_{\text{ref}}$, b) Perform the minimization problem, as outlined in [6], for each of these sets of vectors; c) Identify which choice results in minimum among all of the optimal lengths of the applied average voltage vector in $z_1 - z_2$ plane. With large number of such choices, it is difficult to follow the procedure outlined above.

To find the best technique(s), that results in global minimum length of the applied average voltage vector in $z_1 - z_2$ plane in overmodulation region, a general method is presented in section-III which doesn’t require to decide the switching states beforehand.

III. MINIMIZATION OF HARMONIC INJECTION IN OVERMODULATION

A. Expressing 6φ space-vectors in terms of 3φ space-vectors

Equation (2a) and (2b) apply the standard 3φ − 3φ Clarke’s transformation $C$ on the line-neutral voltages of two 3φ inverters. Here, the prime denotes the quantities related to second inverter. One should note here that $\alpha - \beta$ is 3φ transformed domain and different from 6φ $\alpha - \beta$. With (1), (2a) and (2b), the average voltage vectors in $\alpha - \beta$ and $z_1 - z_2$ planes can be written as the linear combination of average voltage vectors of two three-phase inverters, as shown in (3). Here, $*$ denotes conjugate operation. According to 3φ inverter literature, $\vec{v}_\alpha + j\vec{v}_\beta$ and $\vec{v}_\alpha' + j\vec{v}_\beta'$ are the average voltage space-vectors of the inverter-1 and inverter-2, respectively.

\[
\begin{bmatrix}
  v_\alpha \\
  v_\beta \\
  v_x
\end{bmatrix}
\triangleq
\frac{1}{\sqrt{3}}
\begin{bmatrix}
  1 & -1 & -1 \\
  \frac{-1}{\sqrt{3}} & \frac{2}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\
  1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
  v_{ao} \\
  v_{bo} \\
  v_{co}
\end{bmatrix}
\]  

(2a)

\[
\begin{bmatrix}
  v_\alpha' \\
  v_\beta' \\
  v_x'
\end{bmatrix}^T
\triangleq
C
\begin{bmatrix}
  v_{a'o'} \\
  v_{b'o'} \\
  v_{c'o'}
\end{bmatrix}^T
\]  

(2b)

\[
\vec{v}_{\text{ref}} \triangleq \vec{v}_\alpha + j\vec{v}_\beta = \left(\vec{v}_\alpha + j\vec{v}_\beta\right) e^{j30^\circ}
\]  

(3a)

\[
= (x_1 + x_2) + j(y_1 + y_2)
\]

\[
\vec{v}_{z_1} + j\vec{v}_{z_2} = \left(\vec{v}_\alpha + j\vec{v}_\beta\right) - \left(\vec{v}_\alpha' + j\vec{v}_\beta'\right) e^{j30^\circ}
\]  

(3b)

\[
= (x_1 - x_2) - j(y_1 - y_2)
\]

B. Objective function, equality and inequality constraints

To minimize the applied RMS voltage in $z_1 - z_2$ plane, applied average voltage vector length over each switching cycle needs to be minimized. Therefore, (4) is the objective function which can be derived from (3b). As we want to minimize this objective function for the given $\vec{v}_\alpha$ and $\vec{v}_\beta$, the real and imaginary parts of (3a), i.e., $x_1 + x_2 = \vec{v}_\alpha$ and $y_1 + y_2 = \vec{v}_\beta$, are two equality constraints in this problem.

\[
|x_1 - x_2|^2 + |y_1 - y_2|^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2
\]  

(4)

Fig. 3 shows the mapping of line-neutral voltages generated by 8 switching states (2 zero states and 6 active states) of two-level 3φ inverter into $\alpha - \beta$ in accordance with (2a). Due to balanced load with isolated neutral configuration, voltage generated by these states in $\beta$ plane is zero. By joining the tips of 6 active vectors, a regular hexagon is obtained with length of each side equal to $\frac{V_{DC}}{\sqrt{3}}$. The duty ratios of the six switches of 3φ inverter should be between 0 to 1 and summation of all the duty ratios, corresponding to the switching states applied within a switching cycle, should be equal to 1. With these duty-ratio constraints, the average voltage space-vectors generated by individual 3φ inverter, $\vec{v}_\alpha + j\vec{v}_\beta$ and $\vec{v}_\alpha' + j\vec{v}_\beta'$, will lie within this regular hexagon. As $x_1 + jy_1 = \vec{v}_\alpha + j\vec{v}_\beta$, $x_1 + jy_1 \in B_1$, where $B_1$ is same as hexagon of Fig. 3. As $x_2 + jy_2 \triangleq (\vec{v}_\alpha' + j\vec{v}_\beta') e^{j30^\circ}$, $x_2 + jy_2 \in B_2$, where $B_2$ is obtained by rotating the hexagon of Fig. 3 by $30^\circ$ in anti-clockwise direction. In Fig. 4, $B_1$ and $B_2$ are shown in black and red colours; lengths are normalized with respect to $V_{DC}$. These boundaries of $x_1 + jy_1$ and $x_2 + jy_2$ are the inequality constraints in this problem which are arising due to duty-ratio constraints of the switches of 6φ inverter.
1) Four variables to two variables problem: Substituting $x_2$ and $y_2$ from the two equality constraints in objective function, (4), and two active inequality constraints, (5a) and (5b), the minimization problem becomes as (6), where $\tau_{\alpha}$ and $\tau_{\beta}$ appear as parameters. This optimization problem boils down to determine a point within the feasible region, defined by the inequality constraints of (6), in $x_1 - y_1$ plane so that the distance between that point and the point with co-ordinates $(\frac{\tau_{\alpha}}{2}, \frac{\tau_{\beta}}{2})$ is minimized.

$$\min_{\sqrt{3}x_1 + y_1 \leq 1, x_1 \geq \tau_{\alpha} - \frac{1}{2}} \left( x_1 - \frac{\tau_{\alpha}}{2} \right)^2 + \left( y_1 - \frac{\tau_{\beta}}{2} \right)^2$$

(6)

2) Inequality constraints and search space for optimal solution in $x_1 - y_1$ plane: Fig. 6 helps to visualize this problem geometrically in $x_1 - y_1$ plane. The boundary of the first inequality constraint of (6), i.e., $\sqrt{3}x_1 + y_1 \leq 1$, is shown by $AB$. The boundary of the second inequality constraint, i.e., $x_1 \geq \tau_{\alpha} - \frac{1}{2}$, is shown by $RS$. The $x_1$-intercept of $RS$ varies as $\tau_{\alpha}$ changes. The regions in $x_1 - y_1$ plane, which satisfy the above two inequality constraints, are shown by double-lined arrows on $AB$ and $RS$. $AB$ and $RS$ intersect each other at $E'$. Therefore, $AE'SB$ (shaded region) in Fig. 6, intersection of the above two regions, is the feasible search space for optimal solution.

3) Modified minimization problem in $x_1 - y_1$ plane: As discussed earlier, solving the minimization problem of (6) is equivalent to determine a point within $AE'SB$ so that distance between that point and the point with co-ordinates $(\frac{\tau_{\alpha}}{2}, \frac{\tau_{\beta}}{2})$, suppose $P$, is minimized. As the co-ordinates of both $E'S$ boundary of feasible search space and the point $P$ are functions of $\tau_{\alpha}$ and $\tau_{\beta}$, we will try to trace $E'S$ and $P$ for all possible $(\tau_{\alpha} + j\tau_{\beta}) \in \square IJHG$. The OVM region, $\square IJHG$, can be identified by checking (7).

$$C_1 : \tau_{\alpha} - 1 > 0$$

(7)

Any point within $\square IJHG$ of Fig. 5 has $\tau_{\alpha}$ and $\tau_{\beta}$ in the following range- $1 \leq \tau_{\alpha} \leq \frac{3}{2} + \frac{1}{\sqrt{3}}$ and $0 \leq \tau_{\beta} \leq \frac{\tau_{\alpha}}{\sqrt{3}}$ tan $15^\circ$, for the corresponding $\tau_{\alpha}$. Therefore, $\frac{1}{2} = OD \leq \tau_{\alpha} - \frac{1}{2} = OS \leq \frac{1}{\sqrt{3}} = OB$ is true for all possible $(\tau_{\alpha}, \tau_{\beta}) \in \square IJHG$. Let us consider a line $KL$ parallel to $Y$-axis within $\square IJHG$ such that $KL$ intersects $OH$ and $OG$ at $K$ and $L$, respectively, in Fig. 5. By varying $KL$ from $JL$ to $HG$, we can trace each point within $\square IJHG$. Suppose, we want to solve the optimization problem for a point $(\tau_{\alpha}, \tau_{\beta})$ lying on the line $KL$. Corresponding $(\tau_{\alpha}', \tau_{\beta}')$ will be on $K'L'$ line in Fig. 6, where $OL' = \frac{\tau_{\alpha}}{\sqrt{3}}$. As $K'L' = OL'$ tan $15^\circ$, $K'$ will be on the line $OE$. As $\tau_{\alpha} \geq 1$ for any point within $\square IJHG$, $\frac{1}{2} = OD \leq \tau_{\alpha} - \frac{1}{2} = OS \leq \frac{1}{\sqrt{3}} = OB$. Therefore, $K'L'$ will always be in the left-hand side of $RS$ and right-hand side of $ED$ for $(\tau_{\alpha} + j\tau_{\beta}) \in \square IJHG$.

Let, the line drawn from point $E'$ parallel to $x_1$ axis intersects $K'L'$ at $E''$. The $y_1$ coordinate of $E'$ or $E''$ can be determined by solving the equations of the lines $AB$ and $RS$, which gives $E'S = E''L' = (1 + \frac{\tau_{\alpha}}{\sqrt{3}}) - \sqrt{3}\tau_{\alpha}$. Depending
Fig. 5: Zoomed upper half of sector-1

Fig. 6: Problem in $x_1-y_1$ plane

Fig. 7: Four sub-sectors in sector-1

upon the position of the point with co-ordinates $(\frac{\pi_1}{2}, \frac{\pi_2}{2})$ on $K'L'$, two cases may arise.

**OVM-Case-1:** The point lie on $E''L' \implies \pi_2 \leq E''L'$

$P_1$ is such a point in Fig. 6. As shown in Fig. 6, $Q$ is the
point which is at minimum distance from $P_1$ within feasible region $E'SB$. Therefore, optimal solution, $x_{1,2}^*$ and $y_{1,2}^*$, are
given in (8). With these solutions, optimum $z_1-z_2$ plane
voltages can be determined from (3b) and are given in (8). The
condition $\frac{\pi_1}{2} \leq E''L'$ is equivalent to (9). Equation of the line
passing through $G$ and $J$ in Fig. 5 is $\sqrt{3}X + \frac{\gamma}{2} = (1 + \frac{x_1}{2})$ (shown by dotted line). Therefore, this case arises when tip of
$V_{ref}$ lies within $\Delta GIJ$ of Fig. 5.

$$x_1^* = \pi_1 - \frac{1}{2}; \quad y_1^* = \frac{\pi_2}{2}; \quad x_2^* = \frac{1}{2}; \quad y_2^* = \frac{\pi_2}{2} \quad (8)$$

$$C_2 : \frac{\pi_2}{2} \leq (1 + \sqrt{3}) - \sqrt{3}x_2 \implies \frac{\pi_2}{2} \leq (1 + \sqrt{3}) \quad (9)$$

$$x_1^* = \pi_1 - \frac{1}{2}; \quad y_1^* = (1 + \frac{\sqrt{3}}{2}) - \sqrt{3}x_2 \quad (10)$$

**OVM-Case-2:** The point lie on $K'E'' \implies \frac{\pi_2}{2} > (1 + \frac{x_1}{2}) \implies \text{tip of } V_{ref} \epsilon \Delta GIJ$ of Fig. 5.

$P_2$ is such a point in Fig. 6. In this case, $E'$ is the closest
point from $(\frac{\pi_2}{2}, \frac{\pi_2}{2})$ within the feasible region. The optimal
solution in this case is given in (10).

**E. Minimization in OVM region of lower-half of sector-1**

Similar to upper-half of sector-1, the optimal solution for
OVM region of lower half of sector-1 can be determined. Fig. 7 shows the four overmodulation regions in sector-1 where
**OVM-Case-1** and **OVM-Case-2** in upper-half and lower-
halves are denoted by 1U, 2U and 1L, 2L, respectively. The
inequality through which 2L is differentiated from 1L is given
in (11). The optimal solution in 1L is same as (10) whereas
optimal solution in 2L is given by (12).

$$C_3 : \sqrt{3}x_2 - \frac{\pi_2}{2} > (1 + \frac{\sqrt{3}}{2}) \quad (11)$$

$$x_1^* = \pi_1 - \frac{1}{2}; \quad y_1^* = \sqrt{3}x_2 - (1 + \frac{\sqrt{3}}{2}) \quad (12)$$

As discussed before, the optimal solution in linear region, i.e.,
when $\pi_1 \leq 1$ or tip of $V_{ref} \epsilon \Delta OIJ$ of Fig. 5, can be
determined by plugging $\pi_{z_1} = \pi_{z_2} = 0$ in (3) and given below:

$$x_1^* = x_2^* = \frac{\pi_1}{2}; \quad y_1^* = y_2^* = \frac{\pi_2}{2} \quad (13)$$

IV. OPTIMAL SVOVM TECHNIQUES

The optimal SVOVM techniques of the aforementioned two
OVM cases in upper-half of sector-1 are derived in this section
based on the analysis of the previous section. During selection
of switching sequences, simultaneous switching of two legs of
the 6o inverter and active-vector splitting are not accounted.

**A. Switching Sequences for OVM-Case-1**

1) **Individual inverter state identification:** (8) gives the
optimal solution for this case. As $x_1^* = \frac{1}{2}, \gamma(\frac{\pi_1}{2} + \frac{\pi_2}{2}) \epsilon [\frac{30}{2}] = x_2 + y_2\beta$ lies on line $DC$ of Fig. 5 or $\gamma(\frac{\pi_1}{2} + \frac{\pi_2}{2}) \epsilon [\frac{30}{2}]$ lies on the
side of the hexagon of Fig. 3 which is obtained by joining
states 1 and 6. Therefore, inverter-2 has to apply states 1’
and 6’ and summation of the duty ratios, for which these states
are applied, is 1, i.e., $d_1 + d_6 = 1$. In (8), $\sqrt{3}x_1 + y_1 < 1$.

As $\pi_1 = x_1 \pi_2 = y_1$, according to (3a), $\sqrt{3}\pi_1 + \pi_2 < 1$.
This implies $\pi_1 \epsilon [\pi_1, \pi_2] \epsilon \Delta OAB$ in Fig. 5. We know that
any vector within $30^\circ$ of sector-1 of hexagon of Fig. 3 can
be realized by using either $\{6, 1, 2\}$ or $\{z, 1, 2\}$ states;
where $z$ is zero state and $z \in \{0, 7\}$. Therefore, both $\{6, 1, 2\}$
and $\{z, 1, 2\}$ are possible states of inverter-1 to realize (8).

2) **6o inverter state identification and sequence design:** It
can be shown that $d_1 \geq d_6 \geq d_0 \geq (d_2 + d_4/\beta)$ is true in this
case. Fig. 8a and 8b show the possible states and sequences to
achieve the optimal solution when inverter-1 applies $\{6, 1, 2\}$
and $\{z, 1, 2\}$, respectively. Position of 1’ and 6’ have been kept
fixed and relative positions of states of inverter-1 are changed
in this figure. Fig. 8b is further split into 3 cases depending on
applied zero state: 0, 7 and both 0 and 7 are applied in cases $b_1, b_2$ and $b_3$. Table II tabulates the states and sequences
generated by different cases of Fig. 8. The switching signal
waveforms are symmetric over the sampling period, $T_s$. It can
be seen from table II that both four and five active vectors
can be used to attain optimal solution. Therefore, 8 possible sequences are derived in this case.

### B. Switching Sequences for OVM-Case-2

1) **Individual inverter state identification**: The optimal solution in (10) satisfies $\sqrt{3}x_1 + y_1 = 1$ and $x_2 = \frac{1}{2}$. Therefore, $\pi_{\alpha} + j\pi_{\beta}$ lies on line $AB$ of Fig. 5. As $AB$ is part of the side of the hexagon of Fig. 3, obtained by joining states 1 and 2, therefore (10) can only be realized with states 1 and 2 of inverter-1 with $d_1 + d_2 = 1$ Inverter-2 will apply states 6 and 1, as before.

2) 6$\phi$ inverter state identification and sequence design: It is possible to show that $d_1 \geq d_2 \geq d_0 \geq d_3$ holds in this case. Fig. 6c shows possible 2 sequences where only 3 vectors can be used. Table II lists these sequences.

### C. Derivation of optimal Techniques

Combining the above 8 possible sequences for OVM-Case-1 and 2 sequences for OVM-Case-2, one can have 16 optimal PWM techniques in the whole OVM region.

Existing SOVM4-2 uses case a-I in $\Delta GIJ$ and case c-I in $\Delta GJH$. A carrier comparison based implementation of this optimal technique is discussed in the next section which results into sequence b3-I in $\Delta GIJ$ (2 large, 1 medium and 2 small (2L+1M+2S) active vectors in $\alpha$ - $\beta$ and c-I (2 large, 1 medium (2L+1M) vectors) in $\Delta GJH$. Although all of the techniques of Table II inject optimal low-frequency voltages in $z_1 - z_2$ plane, their high-frequency performances, like, current ripple, flux ripple are expected to differ similar to linear techniques of Table II inject optimal low-frequency voltages in $\alpha$ - $\beta$ and c-I (2 large, 1 medium (2L+1M) vectors).

### D. Average switching frequency in OVM region

Let, the average commutation over one $T_s = \frac{1}{f_s}$ is defined as the ratio of number of legs switch over one $T_s$ and total number of legs (it is 6 for 6$\phi$ inverter). Table II lists the average commutation of different techniques in OVM. For $0.5977 < M \leq 0.5977$, circular $\bar{V}_{ref} = \sqrt{3}Me^{j\omega t}$ lies in $1U$ for $0^\circ \leq \omega t \leq \gamma(M) = \cos^{-1} \frac{1}{\sqrt{3}M}$ and in linear region for $\gamma(M) < \omega t \leq 15^\circ$ in upper-half of sector-1 and $\bar{V}_{ref}$ never enters in $2U$, as shown in Fig. 7 with $M = M_1$. For $0.5977 < M < 0.622$, circular $\bar{V}_{ref}$ lies in $1U$ for $0^\circ \leq \omega t \leq \delta(M) = \sin^{-1} (\frac{2\sqrt{3}}{\sqrt{3}M}) - \tan^{-1} (2\sqrt{3})$ and in $2U$ for $\delta(M) < \omega t \leq 15^\circ$ in upper-half of sector-1 and it never enters in linear region in this modulation index range, as shown in Fig. 7 with $M = M_2$. If conventional SVPWM technique is adopted per 3$\phi$ inverters in linear region where all legs switches, average commutation in linear region is 1. Therefore, average switching frequency, $F_{sw}$, of 6$\phi$ inverter over $15^\circ$ for $0.577 < M \leq 0.5977$ is given by (14) and for $0.5977 < M \leq 0.622$ is given by (15). As modulation over $360^\circ$ is symmetrical for all 6 legs, the average switching frequency over fundamental period is same as (14) and (15).

### V. CARRIER-COMPARISON BASED IMPLEMENTATION OF OPTIMAL TECHNIQUE

Following steps are followed in order to determine the six duty-ratios of top six switches of 6$\phi$ inverter from the given $\pi_{\alpha}, \pi_{\beta}$.

**Step-1 Sector Identification**: Find $k$, so that $30^\circ k - 45^\circ \leq \angle(\pi_{\alpha} + j\pi_{\beta}) \leq 30^\circ k - 15^\circ$

**Step-2 Determination of voltages in modulo-sector**: Find $\pi_{\alpha m} + j\pi_{\beta m} = (\pi_{\alpha} + j\pi_{\beta})e^{-j(k-1)30^\circ}$

**Step-3 Sub-sector Identification**: Use conditions $C_1, C_2$, and $C_3$ of section-III to determine if the modulo vector lies in linear, $2U$, $2L$ or $1U1L$.

**Step-4 Determination of optimal solution**: Using (13) (linear), (8) ($1U$ and $1L$), (10) ($2U$) and (12) ($2L$), find optimal $x_1, y_1, x_2$ and $y_2$. If $k$ is odd, $x_{1m} + jy_{1m} = x_1 + jy_1$; $x_{2m} + jy_{2m} = x_2 + jy_2$. If $k$ is even, $x_{1m} + jy_{1m} = x_2 + jy_2$; $x_{2m} + jy_{2m} = x_1 + jy_1$.

**Step-5 Determination of optimal space-vectors**: The space-vectors corresponding to $k^{th}$ sector can be found as, $\pi_{\theta} + j\pi_{\theta} = (x_{1m} + jy_{1m})e^{j(k-1)30^\circ}$; $\pi_{\alpha} + j\pi_{\beta} = (x_{2m} + jy_{2m})e^{j(k-2)30^\circ}$

**Step-6 Calculation of modulation signals**: Using inverse Clarke’s transformation of (2), $\pi_{\alphao}, \pi_{\betao}, ..., \pi_{c'o}$ can be determined.

**Step-7 Calculation of Six duty ratios**: Duty ratios of the top switches of six legs, $d_\alpha, d_\beta, ..., d_{c'}$, can be calculated by adopting conventional SVPWM technique per 3$\phi$ inverters where $\frac{3}{2} + \frac{m+1}{2}$ common-mode signals to individual 3$\phi$ inverters are injected. This results into application of two zero states ($0$ and $7$ in Table I) for equal duration of time.

14 comparisons, 18 additions and 17 multiplications are required to follow the above steps. As Inverter-1 applies both
TABLE II: Optimal Sequences in OVM (Tip of $V_{ref}$ ∈ □IJHG)

<table>
<thead>
<tr>
<th>OVM Zone</th>
<th>Case</th>
<th>Switching sequence over $T_s$</th>
<th>Vector type</th>
<th>Avg. commutation over $T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1U</td>
<td>a-I</td>
<td>(6, 1') - (1, 1') - (1, 6') - (2, 6') - (1, 0') - (1, 1') - (6, 1')</td>
<td>2L+2M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a-II</td>
<td>(2, 1') - (1, 1') - (1, 0') - (2, 6') - (1, 1') - (1, 1') - (2, 1')</td>
<td>2L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b-I</td>
<td>(1, 1') - (1, 6') - (1, 6') - (0, 6') - (1, 1') - (1, 1') - (0, 1')</td>
<td>2L+1M</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>b-II</td>
<td>(7, 1') - (2, 1') - (1, 1') - (1, 6') - (1, 1') - (2, 1') - (7, 1')</td>
<td>3L+1S</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>b-3-I</td>
<td>(0, 1') - (1, 1') - (1, 6') - (2, 6') - (1, 0') - (1, 1') - (0, 1')</td>
<td>2L+1M+2S</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>b-3-II</td>
<td>(7, 1') - (2, 1') - (1, 1') - (1, 6') - (1, 1') - (2, 1') - (7, 1')</td>
<td>3L+2S</td>
<td></td>
</tr>
<tr>
<td>2U</td>
<td>c-I</td>
<td>(1, 1') - (1, 6') - (2, 6') - (1, 6') - (1, 1') - (1, 1') - (2, 1')</td>
<td>2L+1M</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>c-II</td>
<td>(2, 1') - (1, 1') - (1, 0') - (1, 1') - (2, 1')</td>
<td>3L</td>
<td>2</td>
</tr>
</tbody>
</table>

Experiments and simulations are performed at eleven values of $M$ within $0.577 \leq M \leq 0.622$. Table IV shows the operating condition for each of these experiments. As $V_o = MV_{DC} = 120\sqrt{2}$ V, $V_{DC}$ is changed with $M$ in order to keep $V_o$ and $f_o$ constant for all $M$, although it is customary to keep $V_{DC}$ constant and change $f_o$ with $M$ accordingly. Fig. 11 shows experimental and simulated waveforms of proposed technique. Fig. 11a and 11d show the experimental and simulated waveforms of $i_a$ and $i_a'$, respectively, at $M = 0.612$ which are at a phase difference of $30^\circ$. Similarly, $30^\circ$ phase-shifted line-line voltage waveforms, $v_{ab}$ and $v_{a'b'}$, are shown in Fig. 11c and 11f. $3\phi$ currents of one of the inverters are shown in Fig. 11b and 11e. They are at $120^\circ$ phase difference to each other. The $3\phi$ currents of the other inverter are also similar and they are at $30^\circ$ phase difference to these set of $3\phi$ currents. The experimental line currents and line-line voltages at the maximum boundaries of linear and OVM region, i.e., at $M = 0.577$ and $M = 0.622$ are shown in Fig. 11g, 11j and 11h, 11k, respectively. The line currents are sinusoidal in Fig. 11g. Fig. 11i shows experimental $i_a$ and $i'_b$ at $M = 0.622$ which are sinusoidal, although low-frequency harmonics are present in line-currents. From line-voltage waveforms at three modulation indices, it is clear that switching frequency decreases as $M$ increases.

Fig. 12a and 12d compare analytical and simulated THD and WTHD of proposed technique with all other existing OVM techniques in the range of $0.577 \leq M \leq 0.622$. THD and WTHD of proposed technique is compared with CVPWM experimentally in Fig. 12b and 12e. The continuous lines in

TABLE III: Steady-state Circuit Parameters of $6\phi$ IM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s$, $r_r$</td>
<td>0.675 Ω</td>
</tr>
<tr>
<td>$L_{dss}$, $L_{rss}$</td>
<td>3.75 mH</td>
</tr>
<tr>
<td>$L_{dss}$, $L_{rss}$</td>
<td>0.172 H</td>
</tr>
<tr>
<td>$L_{mss}$</td>
<td>0.168 H</td>
</tr>
</tbody>
</table>

zero states (0 and 7) equally and Inverter-2 doesn’t apply any zero state as the dwell time of zero-state is zero, it results into b3-I in 1U. In 2U, dwell times of zero-states of both of the inverters are zero. But, as all the duty signals are compared with same triangular carrier, it results into c-I.

VI. EXPERIMENTAL AND SIMULATION RESULTS

To validate the proposed optimal technique experimentally, asymmetrical squirrel-cage $6\phi$ induction motor (IM) is chosen as the $6\phi$ machine. The specification of ASPM is 5 kW, 2 pole, 120 V line-neutral RMS in $6\phi$ configuration, 50 Hz. This machine is coupled with 5 kW DC generator which acts as load. The six phases are fed from two $3\phi$ inverters which are built with 1200 V, 75 A SEMIKRON IGBT (SKM75GB123D) module. The controller card consists of Zynq-7010 IC, which has both processing system and programmable hardware, and customized interface card to generate PWM signals for the switches. The experimental set-up is shown in Fig. 10. Table III tabulates the steady-state equivalent circuit parameters of $6\phi$-IM, where winding resistance and leakage inductance of per-phase stator equivalent circuit in $\alpha-\beta$ and $z_1-z_2$ are $r_s$ and $L_{dss}$, respectively; mutual inductance, $L_{m}$, only appears in $\alpha-\beta$. Using these parameters, dynamic model of the machine is implemented in Matlab Simulink to verify the proposed technique through simulation.

TABLE IV: Operating Conditions of Experiments and Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak fundamental line-neutral voltage ($V_o$)</td>
<td>$120\sqrt{2}$ V</td>
</tr>
<tr>
<td>Fundamental frequency ($f_o$)</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Output power</td>
<td>3.5 kW</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>5 kHz</td>
</tr>
</tbody>
</table>
Fig. 11: Experimental and simulated waveforms of proposed technique
these plots correspond to analytical values. The expression of WTHD is given by \( WTHD = \sqrt{\sum_{n=1}^{\infty} \frac{|a_n|}{V_0}} \), which is the indicator of the copper loss incurred in \( z_1 - z_2 \) plane. THD and WTHD performance of proposed 2L1M2S technique is similar to existing SVOVM4-2, as expected from analysis, and better than existing CSVPWM and SVOVM4-1. The close agreement between simulated and experimental results with analysis validates the proposed strategy. Due to dead-time of inverter or slight asymmetry in the winding, experimental THD value at \( M = 0.577 \) is small but non-zero. [21]. Fig. 12c compares simulated low-frequency (harmonic order less than 50) ripple RMS currents of proposed optimal techniques with other existing OVM techniques. As SVOVM4-2 is one of the proposed techniques, it is not shown separately. Fig. 12f compares simulated RMS high-frequency (harmonic order more than 50) ripple currents of 2L2M based SVOVM4-2 and one 4L based optimal technique which uses a-II in \( 1U \) and c-II in \( 2U \) in OVM. The performance of later is better and this justifies our claim that high-frequency ripple current performances of different optimal techniques are different.

VII. CONCLUSIONS AND FUTURE WORK

This paper expresses the average voltage space-vectors of \( 6\phi \) inverter in \( \alpha - \beta \) and \( z_1 - z_2 \) subspaces as a linear combination of average voltage vectors of two \( 3\phi \) inverters. Using these expressions, the paper formulates the problem of minimization of length of the applied average voltage vector of \( z_1 - z_2 \) subspace in each switching cycle in terms of \( 3\phi \) space-vectors. Here, active vectors need not to be specified a priori and hence the formulation is different from the minimization strategy of existing SVOVM techniques of \( 6\phi \) inverter. The proposed formulation achieves the optimal RMS voltage injection throughout the whole overmodulation (OVM) region, i.e., from modulation index 0.577 to 0.622. This paper reports 8 possible ways to realize the optimal strategy in one part of the OVM region and 2 possible ways in another part of OVM region. Each of these ways lead to different PWM techniques which have different current ripple and flux ripple performances. These performance comparisons are not explored in this paper and considered as future work. Finally, an optimal strategy has been identified which can be implemented per \( 3\phi \) inverter basis without involving complex \( 6\phi \) transformation. This technique uses 2 large, 1 medium and 2 small active vectors of \( \alpha - \beta \) in one part of OVM and 2 large, 1 medium vectors in another part. With this strategy, the optimal solution is validated through experiments on laboratory scale hardware prototype and simulations in Matlab at 3.5 kW.

REFERENCES


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