# Analytical Estimation of Turn on Switching Loss of SiC MOSFET and Schottky Diode Pair from Datasheet Parameters 

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#### Abstract

Estimation of switching loss at the early stages of design is essential for determination of switching frequency and selection of power devices. Analytical estimation similar to gate charge method results in fastest and easiest computation when compared with simulation or double pulse test (DPT) based experimental approach. This paper presents an analytical estimation method of turn on switching loss of SiC MOSFET and SiC schottky barrier diode (SBD) pair from datasheet parameters and using values of common source and DC bus inductances. Turn on losses are considered as they dominate the total switching loss. The presented method models the quadratic nature of the transfer characteristics and results in better estimation of current rise time when compared with the linear approximation used in literature. During voltage fall, the non-linear nature of the parasitic capacitances of both the switch and the diode are considered. The simulation and experimental results confirm the accuracy of the presented method over a range of operating conditions for two 1.2 kV discrete SiC MOSFET and SBD pairs of different current ratings.


Index Terms-Analytical switching loss, Double pulse test, Modelling, SiC MOSFET, SiC SBD

## I. Introduction

With excellent switching characteristics and low on state voltage drop, SiC MOSFETs are in close competition with insulated gate bipolar junction transistors (IGBT), which are widely used in voltage range of 900 V to 1700 V [1]-[3]. Objective of this work is find a closed form expression of turn on energy loss of a SiC based Power MOSFET in terms of parameters usually available in the datasheet along with few external circuit related parameters.

There two distinct ways to model a power MOSFET: a) Physics based model, b) Behavioural model. Physics based model is a simulation based approach where the device characteristics are modelled using the theory of device physics and found to be very accurate. For switching loss estimation such a model can be used [4]-[7]. But it requires expensive software like TCAD, domain knowledge, long simulation time and parameters that are usually not available in the datasheet.

Behavioural model is a simplified circuit based model of the MOSFET. Here the three terminal device is modelled with

[^0]non linear voltage dependent current source and a set of non linear voltage dependent capacitors (Fig. 2). For switching loss estimation, this approach overcomes most of the limitations of the physics based model. It is also well known that though simplified, this approach results in quite accurate estimation of switching loss for SiC based power MOSFETS [8]-[11]. Still simulation based on behavioural model for a number of available devices and for a range of operating points is time consuming, particularly at the early stages of design.

In experimental approach, double pulse test (DPT) can be employed to measure switching loss experimentally [3]. This method is expensive and requires difficult high frequency measurements [12]. Moreover it is not possible to measure the actual switching loss due to the parasitics present in the circuit [12]-[14].

Even though simple when compared with the physics based model, the switching dynamics predicted by behavioural model requires solution of a set of coupled non linear differential equations. So no closed form solution for channel current and drain source voltage exists and hence the switching energy loss can not be obtained from this model. The objective of this paper is to develop a closed form expression for turn on energy loss by making appropriate assumptions to the dynamics predicted by the behavioural model through insight and observations from extensive simulation. This approach for analytical estimation of switching loss was adopted in a number of earlier works [15]-[22]. In this paper, we are considering only the turn on loss of discrete 1.2 kV SiC MOSFETs as the turn off loss contributes to a small part of the total switching loss [23].

Turn on switching transition for high voltage MOSFET consists of two sub-parts: current rise and voltage fall. To analyse the switching transients, a buck-chopper configuration is considered as shown in Fig. 1, where $V_{d c}$ is an ideal voltage source. The output inductive load is modelled as a current sink, $I_{0}$. When $v_{g s}<V_{t h}$, MOSFET current $\left(i_{d}\right)$ is zero and it is blocking full DC bus voltage $V_{d c}$. For $v_{g s}>V_{t h}$, $i_{d}$ starts increasing but the voltage $v_{d s}$ remains fixed to $V_{d c}$, as the diode is in conduction (current rise period). After $i_{d}$ becomes $I_{0}, v_{d s}$ starts falling (voltage fall period). The product of $v_{d s}$ and $i_{d}$ becomes non zero during this transition leading to turn on energy loss ( $E_{o n}$ in Joules, the shaded area in Fig. 1(b)). $E_{\text {on }}$ multiplied with switching frequency gives the actual power loss in Watts. For low voltage Si MOSFETs, current rise and voltage fall happens simultaneously [19]-[21]. During current rise, external parasitic inductance effects are
neglected in conventional method such as [15]-[18]. Parasitic common source inductance acts as a negative feedback and slows down current rise. It's effect is considered in [13], [19][22], [24], [25]. But in analysis, the channel current is assumed to be a linear function of gate source voltage. Only exception is [26] where quadratic relation is used but the effect of common source inductance is neglected.


Fig. 1: Piecewise linear turn on transition waveform

During voltage fall, the free wheeling diode contributes to the switching dynamics. In [15]-[18], [25], [27], diode is considered to be ideal while in [13], [19]-[21], a Si based pn junction diode is considered. A SBD based model has been considered in [22], [24] where the reverse capacitance of the diode is taken as constant.

During voltage fall the gate drain capacitance (also known as reverse transfer or miller capacitance) is considered to be constant in [21], [26] whereas two-value ( [13], [17], [18], [24]) and multi-value ( [19], [20], [22], [27]) approximations are used in literature. The drain source capacitance is neglected in [17], [18], [21], [27]. Single value ( [26]), two-value ( [13], [24]) and multi-value approximations ( [19], [20], [22]) of drain source capacitance are known in the literature. Gate charge method during voltage fall is applied in [25].

This work aims to develop an analytical method for turn on loss estimation of SiC MOSFET and SiC schottky diode pair using datasheet parameters, operating conditions and the values of DC bus and common source inductances. The analytical method is based on behavioural model. During current rise, the effect of common source inductance is considered while the channel current is modelled as a quadratic function of gate source voltage. This results in better estimation of current rise time and energy loss. During voltage fall, the nonlinear effect of the parasitic capacitances of both the MOSFET (miller and drain source capacitance) and the schottky diode (reverse bias capacitance) are considered. This results in better estimation of voltage fall time and loss. Consideration of quadratic nature of transfer characteristics and voltage dependent parasitic capacitance model results in a non-linear formulation of the switching dynamics. Unlike Si counterpart, as the switching transitions of SiC MOSFET is much faster, so there is a substantial difference between measured and actual switching loss. This paper presents closed form expressions of both the switching time and actual switching energy loss. The correctness of the implementation of behavioural model is first validated through simulation and experiment for two
devices $(1.2 \mathrm{kV})$ of different current ratings over a range of operating conditions. This also confirms the correctness of parameters read from the datasheet and experimentally obtained external circuit parasitics. Actual loss is estimated through simulation of the the behavioural model and compared with the proposed method. Finally, the proposed method is compared with gate charge, Spice model based simulation and DPT based experimental methods.

In this paper, Section II contains the discussion on device modelling and calculation of switching loss. Section III deals with the analytical switching loss estimation procedure for turn on condition. Details of experimental set-up has been provided in Section IV and simulation and experimental results have been given in section V. Finally, Section VI draws the conclusion.

## II. Device Modelling for Switching Loss Estimation



Fig. 2: Circuit configuration for switching transient analysis
A SiC MOSFET is used as the switch as shown in Fig. 2(a) with three terminals gate $(g)$, drain ( $d$ ) and source $(s)$. A SiC based schottky barrier diode (SBD) is used as the free-wheeling diode (Fig. 2(a)). Performance of SBD is better than the intrinsic body diode of the SiC MOSFET due to it's minimal reverse recovery and low on state drop [2]. Few commercially available SiC MOSFETs are being offered with an anti-parallel SiC based SBD inside the package [28]. Due to the superior characteristics of SBD compared to the intrinsic body diode, it does not play any role in the presence of SBD. So the study will be relevant when applied to a common bidirectional chopper configuration consisting of two such series connected SiC MOSFETs.
$v_{G G}$ is the applied gate driver voltage, which has two levels, $V_{G G}$ and $V_{E E}$ respectively. $R_{g e x t}$ is the total external gate resistance which is the summation of internal resistance of the driver and external gate resistance. $R_{\text {gint }}$ is the internal gate resistance of the MOSFET. $C_{g d(e x t)}$ is the parasitic capacitance seen between the gate $\left(g^{\prime}\right)$ and the drain terminal ( $d$ ) of the device and it depends on the circuit layout. $R_{\text {gint }}$ is connected between the gate lead $\left(g^{\prime}\right)$ and the actual gate terminal $(g)$ of the device, (Fig. 2). The effective gate resistance, $R_{g}$ is the summation of $R_{\text {gext }}$ and $R_{\text {gint }}$.

The equivalent circuit model or behavioural model of the SiC power MOSFET is shown in Fig. 2(b). Shichman-Hodges’
physical model is used to describe the static characteristics [29]. MOSFET is in cut-off region for $v_{g s}<V_{t h}$ and $i_{c h}$ is equal to zero. Here $V_{t h}$ is the threshold voltage of the MOSFET. For $v_{d s}>\left(v_{g s}-V_{t h}\right)$ and $v_{g s}>V_{t h}$, MOSFET is in saturation region and the channel current $i_{c h}$ can be modelled by (1), where $\lambda$ is the channel length modulation index. Using long channel approximation, $\lambda \approx 0$. In ohmic region $v_{d s}<\left(v_{g s}-V_{t h}\right)$ and $v_{g s}>V_{t h}, i_{c h}$ is given by (2).

$$
\begin{equation*}
i_{c h}\left(v_{g s}, v_{d s}\right)=\frac{\beta}{2}\left(v_{g s}-V_{t h}\right)^{2}\left(1+\lambda v_{d s}\right) \approx \frac{\beta}{2}\left(v_{g s}-V_{t h}\right)^{2} \tag{1}
\end{equation*}
$$

$$
\begin{align*}
i_{c h}\left(v_{g s}, v_{d s}\right) & =\frac{\beta}{2}\left(2\left(v_{g s}-V_{t h}\right) v_{d s}-v_{d s}^{2}\right)\left(1+\lambda v_{d s}\right)  \tag{2}\\
& \approx \frac{\beta}{2}\left(2\left(v_{g s}-V_{t h}\right) v_{d s}-v_{d s}^{2}\right)
\end{align*}
$$

Transfer characteristics (in saturation region) of the MOSFET ( $i_{d}$ vs. $v_{g s}$ ) is given in the datasheet for a given temperature. $V_{t h}$ and $\beta$ can be obtained by fitting the transfer characteristics to (1) (Fig. 3). $V_{t h}$ and $\beta$ both are temperature dependent parameters.


Fig. 3: $i_{d}$ vs. $v_{g s}$ curve for C2M0160120D from Wolfspeed, $V_{t h}=4.5 V, \beta=1.08 A / V^{2}$
$C_{g s}, C_{g d}$ and $C_{d s}$ are the gate to source, gate to drain and the drain to source device parasitic capacitances respectively. Input capacitance $C_{i s s}$ is the summation of $C_{g s}$ and $C_{g d}$. Transfer capacitance $C_{r s s}$ is $C_{g d}$ itself. And the output capacitance $C_{o s s}$ is the summation of $C_{d s}$ and $C_{g d}$. In datasheet, $C_{i s s}, C_{r s s}$ and $C_{o s s}$ are plotted as a function of drain source voltage ( $v_{d s}$ ) [30], [31].
$C_{g s}$ is modelled as a constant capacitance. For most of the operating voltage range, $C_{g d}$ is negligible with respect to $C_{g s}$. So $C_{g s}$ will be approximately equal to $C_{i s s} . C_{g d}$ is a non-linear capacitance which depends upon $v_{d s}$. It can be represented as (3) [32]. Similarly, $C_{d s}$ is also a depletion capacitance depends upon $v_{d s}$ and can be modelled as (4).

$$
\begin{align*}
C_{g d}\left(v_{d s}\right) & =\frac{k_{1}}{\left(1+\frac{v_{d s}}{k_{2}}\right)^{1 / 2}+k_{3}}  \tag{3}\\
C_{d s}\left(v_{d s}\right) & =\frac{k_{4}}{\left(1+\frac{v_{d s}}{k_{5}}\right)^{1 / 2}} \tag{4}
\end{align*}
$$

Diode is considered as ideal with zero voltage drop across it during forward biased condition ( $v_{D} \approx 0$ ). In reverse bias,
diode is modelled as a capacitance $C_{D}$, which is also nonlinear function of voltage $\left(v_{D}\right)$ across the diode, (5).

$$
\begin{equation*}
C_{D}\left(v_{D}\right)=\frac{k_{6}}{\left(1+\frac{v_{D}}{k_{7}}\right)^{1 / 2}} \tag{5}
\end{equation*}
$$

$C_{g s}$ is taken same as the $C_{i s s}$ value given in the datasheet for high values of $v_{d s}$. Extraction of parameters $k_{1}$ to $k_{7}$ is done by fitting the equations (3), (4) and (5) to the plots given in the datasheet. $C_{d s}$ vs. $v_{d s}$ plot is not directly given in the datasheet. It can be easily obtained by subtracting $C_{r s s}$ vs $v_{d s}$ plot from the $C_{o s s}$ vs $v_{d s}$ plot. Fig. 4 shows one such example of fitting equation (3) and (4) to the $C_{g d}$ and $C_{d s}$ vs $v_{d s}$ curve taken from the datasheet. Similarly one can obtain $k_{6}$ and $k_{7}$.


Fig. 4: $C_{g d}$ and $C_{d s}$ vs $v_{d s}$ plot for C2M0160120D from Wolfspeed

Due to fast switching transition, external circuit parasitics also play a significant role in switching dynamics. External circuit parasitics that have been considered are the common source inductance, $L_{s}$, part of both the gate and power loop, power loop inductance, $L_{d}$, part of the power circuit loop and external gate to drain capacitance, $C_{g^{\prime} d(e x t)}$, parasitic capacitance between $g^{\prime}$ and $d$ terminal and $C_{a k(e x t)}$, external parasitic capacitance across the diode terminal (Fig. 2(a)). There is also an external parasitic capacitance between $d$ and $s$ terminal of the MOSFET. But the measured value of $C_{d s(e x t)}$ ( $\approx 13.15 \mathrm{nF}$ ) is negligible with respect to the minimum value of the drain to source depletion capacitance, $C_{d s}$ of the MOSFET. $L_{d}$ is the summation of the DC bus inductance, the lead inductances of the MOSFET and the diode and connection inductance between the MOSFET and the diode.

The time evolution of gate source $\left(v_{g s}(t)\right)$ and drain source $\left(v_{d s}(t)\right.$ ) voltage along with the channel current $\left(i_{c h}(t)\right)$ during switching transitions are important for switching loss estimation. Due to the presence of $L_{s}, R_{g i n t}$ and device parasitic capacitances, it is not possible to measure these waveforms experimentally. Instead one can measure $v_{g^{\prime} s^{\prime}}(t), v_{d s^{\prime}}(t)$ and $i_{d}(t)$ (Fig. 2), where $T_{o n}$ is the switching-on transition time. The actual power loss in the MOSFET is given by (6). The measured loss is given by (7).

$$
\begin{align*}
E & =\int_{0}^{T_{o n}} v_{d s}(\tau) i_{c h}(\tau) d \tau  \tag{6}\\
E^{\prime} & =\int_{0}^{T_{o n}} v_{d s^{\prime}}(\tau) i_{D}(\tau) d \tau \tag{7}
\end{align*}
$$

## III. Analytical Estimation of Switching Loss

The objective of this section is to develop closed form expressions to predict the switching loss for given operating conditions $\left(V_{d c}, I_{0}\right)$, gate driver parameters $\left(R_{g e x t}\right.$, $\left.V_{E E}, V_{G G}\right)$ and datasheet parameters of the device $\left(C_{g s}\right.$, $\left.C_{g d}\left(v_{d s}\right), C_{d s}\left(v_{d s}\right), C_{D}\left(v_{D}\right), V_{t h}, \beta, R_{g i n t}\right)$. The external circuit parasitics considered in this analysis is the common source inductance $L_{s}$ and the power loop inductance $L_{d}$. An approximate estimation of $L_{s}$ can be obtained from package information. Generally one should have a rough idea about the $L_{d}$ present in the system. A more accurate value of $L_{d}$ can be obtained through electromagnetic simulation [9]. Effect of $C_{g^{\prime} d(e x t)}$ and $C_{a k(e x t)}$ are not considered to reduce the complexity of the analysis. The variation of parameters with temperature is also not taken into account.

As discussed before, turn on switching loss is a major contributor to the overall switching loss. So the turn on part of the switching transition has been considered here. Turn on switching transition can be divided into five different modes, Mode I to Mode V respectively (Fig. 5).


Fig. 5: Simulation waveforms

## A. Mode I

Mode I is the turn on delay period. In this mode, $v_{g s}$ changes from $V_{E E}$ to $V_{t h}$ and the channel of the SiC MOSFET is not created (inverted). So channel current is zero and device is blocking the DC bus voltage (Mode I in Fig. 5).

## B. Mode II

After the gate source voltage of the SiC MOSFET crosses $V_{t h}$, channel is created. So channel current starts increasing. This is also known as current rise period. In this mode, $i_{d}$ follows $i_{c h}$ and $v_{d s}$ remains almost constant (Mode II in Fig. 5).

We have assumed diode drop $v_{D} \approx 0$. Applying KCL at node $g$, $d$ and $s^{\prime}$ (Fig. 6), we get (8), (9) and (10) correspondingly. Similarly KVL in gate and power loop along with the loop formed by the nodes $g, d$ and $s$ gives us (11), (12) and (13) respectively. For the entire current rise period, the SiC MOSFET is in saturation region. The four state variables of this circuit are $i_{d}, i_{s}, v_{g s}$ and $v_{d s}$. Equations (8) to (13) and (1) completely describe the dynamics of this mode. As (8)


Fig. 6: Equivalent circuit model for Mode II
to (13) and (1) form a set of coupled non-linear differential equations, it can only be solved numerically.

$$
\begin{align*}
i_{g} & =C_{g s} \frac{d v_{g s}}{d t}+C_{g d}\left(v_{d s}\right) \frac{d v_{g d}}{d t}  \tag{8}\\
i_{c h} & =i_{d}+C_{g d}\left(v_{d s}\right) \frac{d v_{g d}}{d t}-C_{d s}\left(v_{d s}\right) \frac{d v_{d s}}{d t}  \tag{9}\\
i_{s} & =i_{d}+i_{g}  \tag{10}\\
V_{G G} & =i_{g} R_{g}+v_{g s}+L_{s} \frac{d i_{s}}{d t}  \tag{11}\\
v_{d s} & =V_{d c}-L_{d} \frac{d i_{d}}{d t}-L_{s} \frac{d i_{s}}{d t}  \tag{12}\\
v_{g s} & =v_{g d}+v_{d s} \tag{13}
\end{align*}
$$

After $v_{g s}$ crosses $V_{t h}$, due to square law, there is a sudden rise in the channel current $\left(i_{c h}\right)$. But due to the presence of $L_{d}$ and $L_{s}$ in power loop, $i_{d}$ can not change momentarily. So an increase in $i_{c h}$ results in the discharge of $C_{g d}$ and $C_{d s}$ (see (9)) or a sharp fall in $v_{d s}$ (initial part of Mode II in Fig. 5). As the rate of change of $i_{d}$ is small initially, the drop across $L_{s}$ is insignificant. After the initial transient, which persists for a small portion of the current rise period, drop across $L_{s}$ becomes significant and slows down the rate of change of $v_{g s}$ (see (11)).

To get a closed form analytical solution of switching loss during current rise period, approximations have been made on the above set of differential equations. As seen from Fig. $5, v_{d s}$ is almost constant during most part of Mode II. So we have assumed $v_{d s}$ constant in this mode. This is a valid assumption for most of the high voltage MOSFETs. $C_{g d}\left(v_{d s}\right)$ and $C_{d s}\left(v_{d s}\right)$ remain almost constant for high value of $v_{d s}$ and the values are small (see Fig. 4). So we have neglected the effect of $C_{d s}\left(v_{d s}\right)\left(d v_{d s} / d t\right)$ in (9) and miller current term $C_{g d}\left(v_{d s}\right)\left(d v_{g d} / d t\right)$ in (8) and (9). In (10), we have neglected $i_{g}$ with respect to $i_{d}$ and $i_{s}$. Then (8) to (13) and (1) can be reduced to (14) and (15).

$$
\begin{align*}
V_{G G} & \approx R_{g} C_{g s} \frac{d v_{g s}}{d t}+v_{g s}+\left(\frac{\beta L_{s}}{2}\right) \frac{d}{d t}\left(v_{g s}-V_{t h}\right)^{2}  \tag{14}\\
v_{d s} & \approx V_{d c}-\left(L_{d}+L_{s}\right) \frac{d i_{c h}}{d t} \\
& \approx V_{d c}-\left(\frac{\beta}{2}\right)\left(L_{d}+L_{s}\right) \frac{d}{d t}\left(v_{g s}-V_{t h}\right)^{2} \tag{15}
\end{align*}
$$

In literature all these assumptions are made along with an additional assumption that variation of channel current $i_{c h}$ is linear with $v_{g s}\left(i_{c h} \approx g_{m}\left(v_{g s}-V_{t h}\right)\right.$, where $g_{m}$ is the
transconductance of the MOSFET). This additional assumption makes (14) a ordinary first order differential equation and $v_{g s}$ rises exponentially with a time constant $\left(R_{g} C_{g s}+g_{m} L_{s}\right)$, $g_{m} L_{s}$ is the additional damping due to the negative feedback of $L_{s}$ that slows down the current rise [25]. In the Appendix we show that such an assumption that makes the solution of (14) straightforward results in significant error. This is also confirmed in the results section.

Though non-linear, (14) can be solved with initial condition $v_{g s}(t=0)=V_{t h}$ and $v_{g s}$ can be written implicitly as a function of time (16). Mode II is completed when $i_{c h}$ reaches $I_{0} . t_{r i}$ is the time period of Mode II or the current rise time. So $i_{c h}\left(t_{r i}\right) \approx I_{0}$, using (1) $v_{g s}\left(t_{r i}\right)=\sqrt{\frac{2 I_{0}}{\beta}}+V_{t h}=V_{m}$. $V_{m}$ is called the miller voltage. Now using (16) with $v_{g s}=V_{m}$, we will get an estimate of $t_{r i}$.

$$
\begin{align*}
t= & \varphi\left(v_{g s}\right) \\
= & -\left(R_{g} C_{g s}+\beta L_{s}\left(V_{G G}-V_{t h}\right)\right) \ln \left(1-\frac{v_{g s}-V_{t h}}{V_{G G}-V_{t h}}\right) \\
& -\beta L_{s}\left(v_{g s}-V_{t h}\right) \tag{16}
\end{align*}
$$

$E_{2}$ in (29) quantifies the loss of energy in this mode. Using (1) and (15), (29) can be simplified to (30). Using change of variable technique and integrating, we have arrived to a closed form expression of $E_{2}$ (31). Where, $d_{1}=$ $-\left(R_{g} C_{g s}+\beta L_{s}\left(V_{G G}-V_{t h}\right)\right), d_{2}=-\beta L_{s}\left(V_{G G}-V_{t h}\right)$ and $d_{3}=\left(\frac{V_{m}-V_{t h}}{V_{G G}-V_{t h}}\right)$

One of the key assumptions is $v_{d s}$ almost constant. This implies $i_{c h}$ must be approximately a linear function of time (from (15)). This can also be seen from the simulation waveforms. Using (16) and (1), we can find out the time evolution of $i_{c h}$. In (16), $R_{g} C_{g s} \ll \beta L_{s}\left(V_{G G}-V_{t h}\right)$ and $\left(\frac{v_{g s}-V_{t h}}{V_{G G}-V_{t h}}\right) \ll 1$ for all practical cases. Neglecting $R_{g} C_{g s}$, approximating the logarithmic term upto second degree polynomial and using (1), we get (17). So $i_{c h}$ rises almost linearly in current rise period. This implies $v_{d s}$ is constant.

$$
\begin{equation*}
i_{c h} \approx \frac{\left(V_{G G}-V_{t h}\right) t}{L_{s}} \tag{17}
\end{equation*}
$$

The slope of $v_{g s}$ or $m$ at the end of current rise period is calculated using (18). Then $v_{d s}$ at the end of the current rise period $\left(V^{*}\right)$, is calculated using (15) and (18) and is given by (19). This quantities $m$ and $V^{*}$ will be used in Mode III and Mode IV.

$$
\begin{align*}
m & =\left.\frac{d v_{g s}(t)}{d t}\right|_{\left(t=t_{r i}\right) \&\left(v_{g s}=V_{m}\right)} \\
& \approx \frac{\left(V_{G G}-V_{m}\right)}{R_{g} C_{g s}+\beta L_{s}\left(V_{m}-V_{t h}\right)}  \tag{18}\\
V^{*} & \approx V_{d c}-\beta m\left(L_{d}+L_{s}\right)\left(V_{m}-V_{t h}\right) \tag{19}
\end{align*}
$$

## C. Mode III

After drain current reaches $I_{0}$, circuit enters into Mode III. This is an intermediate mode between current rise and voltage fall. The SBD is reverse biased and the MOSFET will be in
saturation region throughout this period. So $i_{c h}$ and $v_{g s}$ are related by (1).


Fig. 7: Equivalent circuit model for Mode III and Mode IV

$$
\begin{align*}
v_{D} & \approx V_{d c}-V^{*}-\left(L_{d}+L_{s}\right) \frac{d i_{d}}{d t}  \tag{20}\\
i_{d} & =I_{0}+C_{D}\left(v_{D}\right) \frac{d v_{D}}{d t} \tag{21}
\end{align*}
$$



Fig. 8: Simulation waveforms during intermediate mode
Fig. 7 represents the equivalent circuit for Mode III. As seen from simulation waveforms in Fig. 8, $v_{d s}$ remains almost constant, which is equal to $V^{*}$ and $v_{g s}$ rises almost linearly with slope $m$, so $v_{g s}=\left(m t+V_{m}\right)$. Applying KVL in the power loop (in Fig. 7) and neglecting $i_{g}$ with respect to $i_{d}$ and $i_{s}$, we get (20). KCL at node $d$ gives (21). Using (20) and (21), we get (22), which captures the time evolution of $v_{D}$. For all practical purposes $\left(\frac{v_{D}}{k_{7}}\right) \gg 1$ except for very small values of $v_{D}$. So $C_{D}$ can be approximated by $C_{D} \approx \frac{\alpha_{3}}{\sqrt{v_{D}}}$, where $\alpha_{3}=\left(k_{6} \sqrt{k_{7}}\right)$.

$$
\begin{align*}
v_{D} & \approx V_{d c}-V^{*}-\left(L_{d}+L_{s}\right) \frac{d}{d t}\left(C_{D} \frac{d v_{D}}{d t}\right)  \tag{22}\\
t_{i m} & =2.1\left(\alpha_{3}\left(L_{d}+L_{s}\right)\right)^{0.5}\left(V_{d c}-V^{*}\right)^{-0.25} \tag{23}
\end{align*}
$$

From Fig. 5, this mode ends when $i_{d}$ reaches it's peak or $\frac{d i_{d}}{d t}=0$, implying through (20), $v_{D}\left(t_{i m}\right) \approx\left(V_{d c}-V^{*}\right)$. $t_{i m}$ is the time period of this mode. Though non-linear, it is possible to solve (22) with initial condition $v_{D}(0)=0$ and find $t_{i m}$ from the fact that $v_{D}\left(t_{i m}\right) \approx\left(V_{d c}-V^{*}\right)$, (23). The steps are shown in the Appendix. Switching loss in this period can be estimated using (32). $V_{m}^{*}$ is defined as the approximate gate to source voltage at the end of this mode and is given as $\left(m t_{i m}+V_{m}\right)$.

## D. Mode IV

In this mode $v_{d s}$ falls from $V^{*}$. The MOSFET enters into ohmic region when $v_{d s}=\left(v_{g s}-V_{t h}\right)$. As this voltage is small compared to $V^{*}$, we can assume effectively this mode ends when it enters into ohmic region. This implies during this mode the MOSFET is in saturation region and (1) holds.


Fig. 9: Simulation waveforms during MODE IV
Fig. 7 also represents the equivalent circuit for Mode IV. $v_{g s}$ is approximately constant at $V_{m}^{*}$. So channel current is also constant $\left(\approx\left(\frac{\beta}{2}\right)\left(V_{m}^{*}-V_{t h}\right)^{2}\right)$ and $\frac{d v_{g d}}{d t}=-\frac{d v_{d s}}{d t}$ (see (13)). Drop across $\left(L_{d}+L_{s}\right)$ in power loop is negligible, so $v_{D} \approx\left(V_{d c}-v_{d s}\right)$. For all practical purposes, modulus of $k_{3}$ is close to one and $\left(\frac{v_{d s}}{k_{2}}\right),\left(\frac{v_{d s}}{k_{5}}\right) \gg 1$ except for very small values of $v_{d s}$. So $C_{g d}$ and $C_{d s}$ can be approximated by $C_{g d} \approx \frac{\alpha_{1}}{\sqrt{v_{d s}}}$ and $C_{d s} \approx \frac{\alpha_{2}}{\sqrt{v_{d s}}}$, where $\alpha_{1}=\left(k_{1} \sqrt{k_{2}}\right)$ and $\alpha_{2}=\left(k_{4} \sqrt{k_{5}}\right)$ respectively (Fig. 10). KCL at node $d$ in Fig. 7 gives (24). With the approximations and recognizing $i_{D}=C_{D}\left(v_{D}\right) \frac{d v_{D}}{d t}$, we get (25). This equation completely determines the time evolution of $v_{d s}$ in this mode.

$$
\begin{gather*}
I_{0}+i_{D}+C_{g d}\left(v_{g d}\right) \frac{d v_{g d}}{d t}=i_{c h}+C_{d s}\left(v_{d s}\right) \frac{d v_{d s}}{d t}  \tag{24}\\
\left(\frac{\alpha_{1}+\alpha_{2}}{\sqrt{v_{d s}}}+\frac{\alpha_{3}}{\sqrt{V_{d c}-v_{d s}}}\right) \frac{d v_{d s}}{d t} \approx I_{0}-\left(\frac{\beta}{2}\right)\left(V_{m}^{*}-V_{t h}\right)^{2} \tag{25}
\end{gather*}
$$



Fig. 10: Simplified $C_{g d}$ and $C_{d s}$ vs $v_{d s}$ plot for C2M0160120D from Wolfspeed

Though non-linear, (25) can be solved with the initial condition $v_{d s}(t=0)=V^{*}$ and $v_{d s}$ can be written explicitly
as a function of time (26), where $\psi(t)=h_{1} t+h_{2} . A, B, C$, $D, h_{1}$ and $h_{2}$ are defined below ${ }^{1}$.

$$
\begin{equation*}
v_{d s}(t)=A \psi^{2}(t)+\psi(t) \sqrt{B+C \psi^{2}(t)}+D \tag{26}
\end{equation*}
$$

If $t_{f v}$ is the time period of this mode, then $v_{d s}\left(t_{f v}\right)=$ $\left(V_{m}^{*}-V_{t h}\right)$. Putting this in the expression of $v_{d s}(t)$, first we get a quadratic equation of $\psi^{2}\left(t_{f v}\right)(27)$, where $a, b, c$ and $h_{3}$ are defined below ${ }^{2}$. $t_{f v}$ is calculated using (28). Switching loss in this period is given by (33).

$$
\begin{array}{r}
a \psi^{4}\left(t_{f v}\right)+b \psi^{2}\left(t_{f v}\right)+c=0 \\
t_{f v}=\frac{-\left(h_{2}+\sqrt{h_{3}}\right)}{h_{1}} \tag{28}
\end{array}
$$

After calculating the individual losses in different modes, total loss during turn on period is calculated as follows

$$
E_{a n l y}=E_{2}+E_{3}+E_{4}
$$

## E. Mode V

After the MODE IV is over, $v_{d s}<\left(v_{g s}-V_{t h}\right)$, and the MOSFET enters into ohmic region (equivalent circuit model is shown in Fig. 11). In ohmic region, channel current $i_{c h}$ is


Fig. 11: Equivalent circuit model for Mode V
a function of both gate source voltage $v_{g s}$ and drain source voltage $v_{d s}(2)$. This part of the switching transient is denoted as MODE V. It does not contribute to the turn on switching loss.

In MODE V, $i_{d} \approx i_{c h} \approx I_{0}$ and $\frac{d v_{d s}}{d t} \approx 0$. Applying KCL at $g$ node and KVL in the gate loop, we get (8) and (11) respectively. As $\frac{d v_{d s}}{d t} \approx 0$, then (8) can be approximated as

$$
\begin{aligned}
& { }^{1} A=\frac{\left(\left(\alpha_{1}+\alpha_{2}\right)^{2}-\alpha_{3}^{2}\right)}{\left(\left(\alpha_{1}+\alpha_{2}\right)^{2}+\alpha_{3}^{2}\right)^{2}}, B=\frac{4\left(\alpha_{1}+\alpha_{2}\right)^{2} \alpha_{3}^{2} V_{d c}}{\left(\left(\alpha_{1}+\alpha_{2}\right)^{2}+\alpha_{3}^{2}\right)^{3}}, \\
& C=-\frac{4\left(\alpha_{1}+\alpha_{2}\right)^{2} \alpha_{3}^{2}}{\left(\left(\alpha_{1}+\alpha_{2}\right)^{2}+\alpha_{3}^{2}\right)^{4}}, \quad D=\frac{\alpha_{3}^{2} V_{d c}}{\left(\left(\alpha_{1}+\alpha_{2}\right)^{2}+\alpha_{3}^{2}\right)}, h_{1}= \\
& \frac{1}{2}\left(I_{0}-\frac{\beta}{2}\left(V_{m}^{*}-V_{t h}\right)^{2}\right) \text { and } h_{2}=\left(\alpha_{1}+\alpha_{2}\right) \sqrt{V^{*}}-\alpha_{3} \sqrt{V_{d c}-V^{*}} \\
& 2_{a}^{2}=\left(A^{2}-C\right), \quad b=-\left(B+2 A\left(V_{m}^{*}-V_{t h}-D\right)\right), \quad c= \\
& \left(V_{m}^{*}-V_{t h}-D\right)^{2} \text { and } h_{3}=\frac{-b-\sqrt{b^{2}-4 a c}}{2 a}
\end{aligned}
$$

$$
\begin{gather*}
E_{2}=\int_{0}^{t_{r i}} v_{d s}(\tau) i_{c h}(\tau) d \tau  \tag{29}\\
\approx \int_{0}^{t_{r i}}\left(V_{d c}-\left(L_{d}+L_{s}\right) \frac{d i_{c h}}{d \tau}\right) i_{c h}(\tau) d \tau \\
=\int_{0}^{t_{r i}} V_{d c} i_{c h}(\tau) d \tau-\left.\frac{1}{2}\left(L_{d}+L_{s}\right) i_{c h}^{2}\right|_{i_{c h}(0)} ^{i_{c h}\left(t_{r i}\right)} \\
E_{2}=\frac{\beta V_{d c}}{2} \int_{0}^{t_{r i}}\left(v_{g s}-V_{t h}\right)^{2} d \tau-\left.\frac{\beta^{2}\left(L_{d}+L_{s}\right)}{8}\left(v_{g s}-V_{t h}\right)^{4}\right|_{v_{g s}=V_{t h}} ^{v_{g s}=V_{m}}  \tag{30}\\
E_{2}=\frac{\beta V_{d c}}{2} \int_{v_{g s}=V_{t h}}^{V_{m}}\left(v_{g s}-V_{t h}\right)^{2}\left(\frac{d \varphi\left(v_{g s}\right)}{d v_{g s}}\right) d v_{g s}-\frac{\beta^{2}\left(L_{d}+L_{s}\right)}{8}\left(V_{m}-V_{t h}\right)^{4} \\
=\frac{\beta V_{d c}}{2}\left(V_{G G}-V_{t h}\right)^{2}\left(d_{1}\left(d_{3}+\frac{d_{3}^{2}}{2}+\ln \left(1-d_{3}\right)\right)+\frac{d_{2} d_{3}^{3}}{3}\right)-\frac{\beta^{2}\left(L_{d}+L_{s}\right)}{8}\left(V_{G G}-V_{t h}\right)^{4} d_{3}^{4} \tag{31}
\end{gather*}
$$

$$
\begin{align*}
E_{3} & =\int_{0}^{t_{i m}} v_{d s}(\tau) i_{c h}(\tau) d \tau \\
& =\left(\frac{\beta}{2}\right) \int_{0}^{t_{i m}} V^{*}\left(m \tau+\left(V_{m}-V_{t h}\right)\right)^{2} d \tau \\
& =\frac{\beta V^{*}}{6 m}\left(\left(V_{m}^{*}-V_{t h}\right)^{3}-\left(V_{m}-V_{t h}\right)^{3}\right)  \tag{32}\\
E_{4} & =\int_{0}^{t_{f v}} v_{d s}(\tau) i_{c h}(\tau) d \tau \\
& =\left(\frac{\beta}{2}\right)\left(V_{m}^{*}-V_{t h}\right)^{2} \int_{0}^{t_{f v}} v_{d s}(\tau) d \tau \\
& =\left(\frac{\beta}{2}\right)\left(V_{m}^{*}-V_{t h}\right)^{2}\left(\frac{A\left(\psi\left(t_{f v}\right)^{3}-\psi(0)^{3}\right)}{3 h_{1}}+\frac{\left(B+C \psi^{2}\left(t_{f v}\right)\right)^{3 / 2}-\left(B+C \psi^{2}(0)\right)^{3 / 2}}{3 C h_{1}}+D t_{f v}\right) \tag{33}
\end{align*}
$$

(34), where $C_{i s s}=C_{g s}+C_{g d}$. Using (11) and (34), we get a differential equation of $v_{g s}$ (35).

$$
\begin{align*}
i_{g} & \approx C_{i s s} \frac{d v_{g s}}{d t}  \tag{34}\\
V_{G G} & \approx R_{g} C_{i s s} \frac{d v_{g s}}{d t}+v_{g s}+L_{s} C_{i s s} \frac{d^{2} v_{g s}}{d t^{2}} \tag{35}
\end{align*}
$$

For our application, values of $R_{g}, L_{s}$ and $C_{i s s}$ are such that the response will be over damped. As $v_{g s}(\infty)=V_{G G}$ and $\left(V_{G G}-V_{t h}\right)>v_{d s}$, then using (2), channel resistance $R_{c h}$ at fully on condition can be approximated as (36). But there is another component of resistance, which is called drift resistance. So on state resistance is the summation of channel resistance and drift resistance.

$$
\begin{equation*}
R_{c h} \approx \frac{1}{\beta\left(V_{G G}-V_{t h}\right)} \tag{36}
\end{equation*}
$$

## IV. Experimental Set-up



Fig. 12: Double pulse test setup
Double pulse test (DPT) has been conducted to validate the behavioural model. It also confirms that the device related parameters are correctly extracted from the datasheet for a given diode switch pair and used in simulation and analytical

TABLE I: Device list

| MOSFET | Current (A) <br> $@ 100^{\circ} \mathrm{C}$ | Diode | Current (A) <br> $@ 135^{\circ} \mathrm{C}$ |  |
| :--- | :---: | :---: | :--- | :---: | :---: |
| SET1 | C2M0160120D | 12.5 | C4D05120A | 9.5 |
| SET2 | C2M0080120D | 24 | C4D10120A | 16 |

computation. It helps us to establish one of the conclusions of this work that there is a significant difference between the experimentally obtained loss and actual switching loss.

To validate our model through experiment, two sets of SiC MOSFET and SiC SBD diode pairs (TABLE I) have been tested. i.e SET1 represents SiC MOSFET C2M0160120D and SiC SBD diode C4D05120A. These are all discrete devices with package type of TO-247 and TO-220 for MOSFET and diode respectively. All the devices are 1200 V rated. Device related parameters extracted from the datasheet are given in TABLE II (for junction temperature $T_{j}$ of $25^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$ ). Among them only $V_{t h}$ and $\beta$ are strong function of $T_{j}$ and their temperature variations are taken into consideration.

SiC MOSFET requires bipolar gate voltage of $+20 /-$ 5V. Opto-isolator IX3180GS followed by a current booster IXDN609SI is used to drive the gate of the SiC MOSFET. Gate driver parameters used in simulation and experiment are same for both the sets and are given in TABLE III.

DPT is designed for 800 V DC bus and 30A load current. Film capacitor from EPCOS (B32776G1805+000) with $8 \mu \mathrm{~F}$ capacitance and 1300 V DC blocking capability is used. Eight such capacitors are connected in parallel to achieve the net capacitance of $64 \mu \mathrm{~F}$, that supports the switching current and minimize the DC bus inductance. Air core inductor with 150 $\mu \mathrm{H}$ inductance is used for the output inductive load. The values of circuit related parameters used in simulation and analytical loss estimation are given as follows: $L_{d}=45 \mathrm{nH}$, $L_{s}=6.5 n H$ and $C_{g^{\prime} d(e x t)}=15 p F=C_{a k(e x t)} . L_{d}$ and $L_{s}$ depends on the package type as well as layout and the values are same for both SET1 and SET2. On the other hand, $C_{g^{\prime} d(e x t)}$ and $C_{a k(e x t)}$ depends solely on PCB layout. For experimental determination of $C_{g^{\prime} d(e x t)}$, the MOSFET is disconnected and a large resistance is connected in series. Finally the step response of the resultant circuit is observed. As $g^{\prime}, d$ and $a, k$ are the adjacent pins of the MOSFET and SBD respectively, the layouts are almost same. So the values of $C_{g^{\prime} d(e x t)}$ and $C_{a k(e x t)}$ come close. For each diode switch pair (SET1, 2), experiments are conducted for two values of $V_{d c}$, four values of $I_{0}$ and three values of $R_{g e x t}$. This implies total 48 different operating conditions where values of the circuit related parameters remain same. The values of these parameters are obtained through experiment and simulation at one of these operating conditions.

1 GHz oscilloscope (MDO3104) from Tektronix is used for measurement. Signals need to be measured are $v_{g^{\prime} s^{\prime}}(t), v_{d s^{\prime}}(t)$ and $i_{d}(t)$. A passive probe from Tektronix with 1 GHz bandwidth (TPP1000) is used for $v_{g^{\prime} s^{\prime}}(t)$ measurement. For $v_{d s^{\prime}}(t)$, a high voltage single ended probe from Tektronix (P5100A) with 500 MHz bandwidth is used. Current $i_{d}(t)$ is measured using a AC/DC current probe from Tektronix (TCP0030A)
with 120 MHz bandwidth and 50A peak current measurement capability. Matching of propagation delay between voltage and current signals are important when signals within few nanoseconds are measured. The delay in current probes are usually higher than the delay in voltage probes. So to match the delay between voltage and current probes, we used a delay matching instrument available from Tektronix (067-1686-00, Power Measurement Deskew and Calibration Fixture).


Fig. 13: SET1: Operating conditions: [800V, $7.5 \Omega, 10 \mathrm{~A}]$ (a) Simulation waveforms, (b) Experimental waveforms: $i_{d}$ (3A/div.), $v_{d s^{\prime}}(200 \mathrm{~V} /$ div. $), v_{g^{\prime} s^{\prime}}(15 \mathrm{~V} /$ div. $)$ and $t(10 \mathrm{~ns} /$ div. $)$


Fig. 14: SET2: Operating conditions: [800V, $7.5 \Omega, 20 \mathrm{~A}]$ (a) Simulation waveforms, (b) Experimental waveforms: $i_{d}$ ( $5 \mathrm{~A} /$ div. $), v_{d s^{\prime}}(200 \mathrm{~V} /$ div. $), v_{g^{\prime} s^{\prime}}(20 \mathrm{~V} /$ div. $)$ and $t(10 \mathrm{~ns} /$ div. $)$

To simulate the circuit shown in Fig. 2(a), we have used the circuit simulator called SimPowerSystems that works in the Simulink environment of MATLAB. To implement the behavioural model of the MOSFET (Fig. 2(b)) and also the SBD, we used voltage dependent current sources and variable capacitances available in SimPowerSystems library. All the experiments are performed at $25^{\circ} \mathrm{C}$.

## V. Simulation and Experimental Results

The objective of this section is to validate the analytical loss estimation method presented in Section III through simulation and experiment.

## A. Validation of the behavioural model

$v_{g^{\prime} s^{\prime}}(t), v_{d s^{\prime}}(t)$ and $i_{d}(t)$, the key waveforms related to turn on transition and shown in Fig 13 and Fig 14 for SET1 and SET2 respectively. The operating conditions are $V_{d c}=800 \mathrm{~V}, R_{g e x t}=8.5 \Omega$ and $I_{0}=10 \mathrm{~A}$ for SET1 and $I_{0}=20 A$ for SET2. Experimental waveforms matches closely with simulation over the switching transition period. In Fig. 15 simulation and experimental results are plotted in the same plot for $V_{d c}=800 \mathrm{~V}, R_{\text {gext }}=8.5 \Omega$ and four different current levels $\left(I_{0}\right)$ for SET1 and SET2. This observation is seen to hold for other 40 operating conditions also. After the transition is over, there is an oscillation observed in $v_{d s^{\prime}}$ which is not predicted by the behavioural

TABLE II: Device parameters

| $V_{t h}(\mathrm{~V})$ |  | $\beta\left(\mathrm{A} / V^{2}\right)$ | $R_{g i n t}$ <br> $(\Omega)$ | $C_{g s}$ <br> $(\mathrm{nF})$ | $k_{1}$ <br> $(\mathrm{nF})$ | $k_{2}$ <br> $(\mathrm{~V})$ | $k_{3}$ | $k_{4}$ <br> $(\mathrm{nF})$ | $k_{5}$ <br> $(\mathrm{~V})$ | $k_{6}$ <br> $(\mathrm{nF})$ | $k_{7}$ <br> $(\mathrm{~V})$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $25^{\circ} C$ | $100^{\circ} C$ | $25^{\circ} C$ | $100^{\circ} C$ |  |  |  |  |  |  |  |  |  |
| SET1 | 4.5 | 2.75 | 1.08 | 0.9 | 6.5 | 0.525 | 0.04 | 4 | -0.85 | 0.43 | 5.5 | 0.39 | 1.5 |
| SET2 | 5.5 | 3.75 | 1.4 | 1.3 | 4.6 | 0.95 | 0.095 | 4 | -0.7625 | 1.1 | 2.6 | 0.754 | 1.7 |



Fig. 15: Simulation vs experimental waveforms, Operating condition: [800V, $8.5 \Omega$ ]: (a) SET1, (b) SET2

TABLE III: Driver parameters

| $V_{C C}$ <br> $(\mathrm{~V})$ | $V_{G G}$ <br> $(\mathrm{~V})$ | $R_{g(\text { driver })}$ <br> $(\Omega)$ | $R_{\text {gext }}$ <br> $(\Omega)$ |
| :---: | :---: | :---: | :---: |
| -5 | 20 | 1 | $2.5,4.5,8.5$ |

model. This results in post switching ringing loss and it is not accounted in this work. After that, measured loss computed from simulation ( $E_{\text {sim }}^{\prime}$ ) (using (7)) is compared with the measured loss from the experiment ( $E_{\text {exp }}^{\prime}$ ) (using (7)) for the entire operating range ( 48 different operating conditions) and found to be closely matching (see TABLE IV). Also important intermediate quantities are compared between experimental and simulation results for $V_{d c}=800 \mathrm{~V}, R_{\text {gext }}=2.5 \Omega$ and $I_{0}=10 A$ for SET1 and $I_{0}=20 A$ for SET2 (see TABLE V). A close agreement is observed.

These set of results confirm the following: behavioural model is accurate enough to predict the switching transient and device and circuit related parameters are correctly extracted and used in simulation.


Fig. 16: Intermediate quantity verification: (a) MODE III, (b) MODE IV

## B. Verification of analytical loss estimation

Important intermediate quantities for analytical loss estimation (see Section III) are computed for SET1 and SET2. The operating conditions are $V_{d c}=800 \mathrm{~V}, R_{\text {gext }}=2.5 \Omega$ and $I_{0}=10 \mathrm{~A}$ for SET1 and $I_{0}=20 \mathrm{~A}$ for SET2. These numbers are compared with the numbers obtained from simulation (TABLE VI). As the numbers match closely, this validates the analysis in each mode of switching transition. Rate of change of $i_{d}$ during current rise period is computed using (17) and it is closely matching with the number obtained from the simulation. It verifies that $i_{c h}$ changes linearly during current rise period and $v_{d s}$ remains constant. Current rise time $\left(t_{r i}\right)$ has been estimated using the method described in [25] for SET1 for the operating condition $V_{d c}=800 \mathrm{~V}, R_{\text {gext }}=2.5 \Omega$ and $I_{0}=10 A$ (transconductance $g_{m}=4.8 S$ is taken) and comes out to be 3.04 ns . So the error in estimation is around $55 \%$. Also during MODE III and MODE IV, two of the important state variables are $v_{D}(t)$ and $v_{d s}(t)$ respectively. To verify the assumptions made in each mode, analytically obtained state variables are plotted along with simulated result for SET1 and for the previously mentioned operating conditions (Fig. 16) ( $v_{D}(t)$ for MODE III (using (40)) and $v_{d s}(t)$ for MODE IV (using (26))).

Actual loss computed from simulation ( $E_{\text {sim }}$ ) (see (6)) is compared with analytical loss $E_{\text {anly }}$ (as described in section III) over the operating range (Fig. 17 and Fig. 18). In Fig. 17(a), $E_{a n l y}$ is computed for SET1 and plotted for $V_{d c}=$ 800 V and $600 \mathrm{~V}, R_{\text {gext }}=2.5 \Omega, T_{j}=25^{\circ} \mathrm{C}$ and in the current range of $I_{0}=(2.5,10) A$. $E_{s i m}$ is also computed from simulation using (6) for four different current values and plotted as points on the same plot. The percentage of error between $E_{\text {anly }}$ and $E_{\text {sim }}$ is shown in a bar diagram below for


Fig. 17: SET1: Actual loss Comparison: Analytical vs Simulation: (a) $T_{j}=25^{\circ} \mathrm{C}$, (b) $T_{j}=100^{\circ} \mathrm{C}$


Fig. 18: SET2: Actual loss Comparison: Analytical vs Simulation: (a) $T_{j}=25^{\circ} \mathrm{C}$, (b) $T_{j}=100^{\circ} \mathrm{C}$

TABLE IV: Comparison between $E_{s i m}^{\prime}$ and $E_{\text {exp }}^{\prime}\left(I_{0}\right.$ in A and $E^{\prime} s$ are in $\left.\mu J\right)$

|  |  | $[600 \mathrm{~V}, 2.5 \Omega]$ | $[600 \mathrm{~V}, 4.5 \Omega]$ | $[600 \mathrm{~V}, 8.5 \Omega]$ | $[800 \mathrm{~V}, 2.5 \Omega]$ | $[800 \mathrm{~V}, 4.5 \Omega]$ | $[800 \mathrm{~V}, 8.5 \Omega]$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $I_{0}$ | $E_{\text {sim }}^{\prime}$ | $E_{\text {exp }}^{\prime}$ | $E_{\text {sim }}^{\prime}$ | $E_{\text {exp }}^{\prime}$ | $E_{\text {sim }}^{\prime}$ | $E_{\text {exp }}^{\prime}$ | $E_{\text {sim }}^{\prime}$ | $E_{\text {exp }}^{\prime}$ | $E_{\text {sim }}^{\prime}$ | $E_{\text {exp }}^{\prime}$ | $E_{\text {sim }}^{\prime}$ | $E_{\text {exp }}^{\prime}$ |
| SET1 | 2.5 | 19.2 | 19.25 | 20.41 | 20.44 | 22.81 | 22.07 | 30.88 | 29.6 | 32.89 | 30.8 | 36.92 | 36.3 |
|  | 5 | 30 | 28.55 | 32.39 | 31.29 | 36.85 | 38.8 | 47.39 | 44.96 | 51.25 | 49.34 | 59.45 | 58.11 |
|  | 10 | 61.21 | 57.34 | 65.42 | 64.83 | 74.2 | 85.36 | 94.05 | 63 | 73.95 | 71.95 | 86.4 | 87.9 |
|  | 5 | 46.96 | 41.76 | 50.43 | 46.7 | 57.3 | 56.09 | 73.15 | 60.52 | 78.68 | 73.62 | 90.6 | 89.32 |
| SET2 | 10 | 88.49 | 73 | 94.7 | 82 | 108.7 | 106.34 | 134.83 | 116 | 145.2 | 132.22 | 169 | 169.2 |
|  | 15 | 142.74 | 129 | 152.51 | 144.6 | 175.02 | 165.45 | 213.34 | 186.58 | 230.08 | 226.95 | 269 | 262.4 |
|  | 20 | 210.7 | 205.35 | 225 | 211.15 | 257.93 | 238.86 | 312 | 308.9 | 336.08 | 337.42 | 392.9 | 380.5 |

TABLE V: Intermediate quantities comparison (Experiment and Simulation)

|  |  | Current rise |  |  |  |  | Voltage fall |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $t_{r i}^{\prime}$ <br> $(\mathrm{ns})$ | $\frac{d i_{d}}{d t}$ <br> $(\mathrm{~A} / \mathrm{ns})$ | $V^{* *}$ <br> $(\mathrm{~V})$ | $E_{r i}^{\prime}$ <br> $(\mu \mathrm{J})$ | $t_{f v}^{\prime}$ <br> $(\mathrm{ns})$ | $E_{f v}^{\prime}$ <br> $(\mu \mathrm{J})$ | $E^{\prime}$ <br> $(\mu \mathrm{J})$ |  |
| SET1 | Sim | 5.8 | 2.5 | 730 | 17.4 | 13.7 | 76.6 | 94 |
|  | Exp | 6.2 | 2.5 | 690 | 15.2 | 15.2 | 71.8 | 89 |
| SET2 | Sim | 14.8 | 1.9 | 746 | 109.2 | 18.6 | 202.8 | 312 |
|  | Exp | 12.6 | 2 | 750 | 102 | 21.6 | 206.9 | 308.9 |

four different current values. Same procedure has been carried out for $R_{\text {gext }}=4.5 \Omega$ and $R_{\text {gext }}=8.5 \Omega$ (Fig. 17(a)). Similar comparison has also been done for SET1 at $T_{j}=100^{\circ} \mathrm{C}$ (see Fig. 17(b)) and SET2 for both $T_{j}=25^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$ (shown in Fig. 18).

The observations made from the set of results of Fig. 17 and Fig. 18 are following: Maximum percentage of error between $E_{\text {sim }}$ and $E_{\text {anly }}$ is around $21 \%$ for both SET1 and SET2. For low value of $R_{\text {gext }}, E_{\text {anly }}$ matches with $E_{\text {anly }}$ in close agreement (min. percentage of error is around $0.4 \%$ ). But as $R_{\text {gext }}$ increases, percentage of error also goes up. For low $V_{d c}$, there is a better match between $E_{\text {sim }}$ and $E_{a n l y}$ for most of the operating conditions. Also for a given operating condition, with increase in junction temperature $\left(T_{j}\right)$ turn on loss reduces and the variation over a range from $25^{\circ} \mathrm{C}$ to $100^{\circ} \mathrm{C}$ is also not significant. This was also concluded in [33].

## C. Comparison between proposed loss estimation method and other existing methods

Analytically estimated loss $\left(E_{\text {anly }}\right)$ is compared with conventional gate charge method ( $E_{G C}$ ) [15], [16], experimentally obtained loss $\left(E_{\text {exp }}^{\prime}\right)$ and loss obtained from the Spice model ( $E_{\text {spice }}^{\prime}$ ) in this subsection with respect to actual loss from simulation $\left(E_{\text {sim }}\right)$ for $V_{d c}=600 \mathrm{~V}$ and $R_{g e x t}=2.5 \Omega$ (Fig. 19 ) and $V_{d c}=800 V$ and $R_{g e x t}=8.5 \Omega$ (Fig. 20). The Spice Model of the MOSFET gives a four terminal block, where power terminals are $g^{\prime}, d$ and $s^{\prime}$, Fig. 2. The common source inductance $L_{s}$ and the internal gate resistances $\left(R_{\text {gint }}\right)$ are already modelled inside the block. We changed the value of $L_{s}$ to 6.5 nH in the parameter list. The other terminal is an input for junction temperature $\left(T_{j}\right)$, that we have set at $25^{\circ} \mathrm{C}$.

Analytical method performs better than gate charge method, experimental approach and Spice model based simulation
approach over the entire range. Also experimental approach performs close to spice based simulation approach and both of these approaches perform better over gate charge method for most of the operating range.


Fig. 19: Comparison with experiment, gate charge method [15], [16] and SPICE model for $V_{d c}=600 \mathrm{~V}$ and $R_{\text {gext }}=$ $2.5 \Omega$


Fig. 20: Comparison with experiment, gate charge method [15], [16] and SPICE model for $V_{d c}=800 \mathrm{~V}$ and $R_{g e x t}=$ $8.5 \Omega$

As we increase the value of $R_{g e x t}$, the error between $E_{\text {sim }}$ and $E_{a n l y}$ also increases (see Fig. 17 and Fig. 18). Also $R_{\text {gext }}$ slows down the voltage fall transition which reduces the difference between $E_{\text {exp }}^{\prime}$ and $E_{\text {sim }}$.

## VI. Conclusion

In this paper an analytical method for the estimation of turn on switching loss of SiC MOSFET and SiC schottky diode pair is presented. This method is based on the behavioural model of the devices. The proposed method provides closed form expressions of loss in terms of device parameters from datasheet and parasitic inductances while considering nonlinear nature of the transfer characteristics and the parasitic capacitances. In addition to the current rise and voltage fall modes seen in regular high voltage MOSFETs, an intermediate mode has been observed during switching transition.

Proposed method has been verified experimentally for two sets of devices (SiC MOSFET and SiC SBD) of different

TABLE VI: Comparison of important intermediate quantities (Simulation and Analytical)

|  |  | Mode II |  |  |  |  | Mode III |  |  | Mode IV |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} t_{r i} \\ (\mathrm{~ns}) \end{gathered}$ | $\begin{gathered} E_{2} \\ (\mu \mathrm{~J}) \end{gathered}$ | $\frac{d i_{d}}{d t}$ | $V^{*}$ <br> (V) | $\begin{gathered} m \\ (\mathrm{~V} / \mathrm{ns}) \end{gathered}$ | $t_{i m}$ <br> (ns) | $\begin{gathered} E_{3} \\ (\mu \mathrm{~J}) \end{gathered}$ | $\begin{aligned} & V_{m}^{*} \\ & (\mathrm{~V}) \end{aligned}$ | $t_{f v}$ <br> (ns) | $\begin{gathered} E_{4} \\ (\mu \mathrm{~J}) \end{gathered}$ | $\begin{gathered} E \\ (\mu \mathrm{~J}) \end{gathered}$ |
| SET1 | Simulation | 6.8 | 25.18 | 2.5 | 729.5 | 0.325 | 4.3 | 42 | 10.3 | 8.7 | 52 | 119 |
|  | Analytical | 6.5 | 22.5 | 2.3 | 720 | 0.34 | 3.8 | 35.53 | 10.1 | 10.1 | 51 | 109 |
| SET2 | Simulation | 15.6 | 118.19 | 1.9 | 740 | 0.175 | 6.1 | 105 | 11.95 | 15.1 | 131 | 354.65 |
|  | Analytical | 15.58 | 113.2 | 2.3 | 737 | 0.1625 | 5.42 | 92 | 11.71 | 18.18 | 159 | 364 |

current ratings for a large set of operating conditions (DC bus voltage, load current and gate resistance). The correctness of the implementation of behavioural model is validated through Spice based simulation and experiment. It also validates the correctness of the data read from the datasheet and the measured external circuit parasitics. It has been established that there is a significant difference between actual and measured switching loss. The proposed analytical loss closely matches with the actual loss obtained through simulation with percentage error that lies between 0.4 to $21 \%$ over a wide range of operating conditions. Also the proposed loss estimation technique has been verified for two different junction temperature ( $T_{j}=25^{\circ} \mathrm{C}$ and $T_{j}=100^{\circ} \mathrm{C}$ ). It has been found that For a given operating condition, with increase in temperature turn on loss reduces and the variation over a range from $T_{j}=25^{\circ} \mathrm{C}$ to $T_{j}=100^{\circ} \mathrm{C}$ is also not significant. The proposed method performs better than conventional gate charge method and Spice based simulation method over the entire operating range while the performance of the double pulse test based experiment is comparable only at high values of gate resistance.

## VII. Appendix

A. Error introduced by considering $i_{c h}$ as a linear function of $v_{g s}$

In literature, $i_{c h}=\left(\frac{\beta}{2}\right)\left(v_{g s}-V_{t h}\right)^{2}$ is approximated by $\tilde{i_{c h}}=g_{m}\left(v_{g s}-V_{t h}\right)$ where $g_{m}=\left.\frac{d i_{d}}{d v_{g s}}\right|_{\left(v_{g s}=V_{m}\right)}=\beta\left(V_{m}-\right.$ $\left.V_{t h}\right)$. So the error $\left|\frac{i_{c h}-\tilde{i_{c h}}}{i_{c h}} \times 100\right|_{\left(v_{g s}=V_{m}\right)}$ is $100 \%$.

## B. Solution of non-linear differential equation of Mode III

(22) can be written as (37), where $d_{4}=0.5 \alpha_{3}\left(L_{d}+L_{s}\right)$. Replacing $y=\left(\frac{d v_{D}}{d t}\right)$ in (37), we get (38). After that, putting $y^{2}=z$, we get a differential equation (39).

$$
\begin{align*}
2 d_{4} \frac{d^{2} v_{D}}{d t^{2}}-\frac{d_{4}}{v_{D}}\left(\frac{d v_{D}}{d t}\right)^{2} & =\left(V_{d c}-V^{*}-v_{D}\right) v_{D}^{1 / 2}  \tag{37}\\
2 d_{4} y \frac{d y}{d v_{D}}-\frac{d_{4}}{v_{D}} y^{2} & =\left(V_{d c}-V^{*}-v_{D}\right) v_{D}^{1 / 2}  \tag{38}\\
\frac{d z}{d v_{D}}-\frac{z}{v_{D}} & =\frac{\left(V_{d c}-V^{*}-v_{D}\right)}{d_{4}} v_{D}^{1 / 2} \tag{39}
\end{align*}
$$

Solving it, we get (40). $t_{i m}$ is the the value of $t$ when $v_{D}$ reaches $\left(V_{d c}-V^{*}\right)$. Integrating (40), we get (41) where $B_{1 / 3}(1 / 4,1 / 2)$ is an incomplete beta function. From table $B_{1 / 3}(1 / 4,1 / 2)=3.2$. So (41) can be written as (23).

$$
\begin{align*}
z & =\frac{2\left(V_{d c}-V^{*}\right) v_{D}^{3 / 2}-\frac{2}{3} v_{D}^{5 / 2}}{d_{4}}  \tag{40}\\
t_{i m} & =d_{4}^{1 / 2} \int_{0}^{V_{d c}-V^{*}} \frac{d v_{D}}{\sqrt{2\left(V_{d c}-V^{*}\right) v_{D}^{3 / 2}-\frac{2}{3} v_{D}^{5 / 2}}} \\
t_{i m} & =\frac{3^{0.25}}{2^{0.5}} d_{4}^{0.5}\left(V_{d c}-V^{*}\right)^{-0.25} B_{1 / 3}(1 / 4,1 / 2) \tag{41}
\end{align*}
$$

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