# A New Space-vector PWM Technique of Two-level Inverter Fed Asymmetrical Six-phase Machine: Analysis and Performance Evaluation 

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#### Abstract

Asymmetrical six-phase machine (ASPM) with six balanced phases and two isolated neutral points can be analyzed in two orthogonal two-dimensional subspaces where one of them is associated with electromechanical energy transfer. Excitation of the non-energy transferring plane causes unwanted copper loss. Therefore, linear modulation techniques (LMTs) of inverter fed ASPM synthesize the desired reference voltage vector in the energy transferring plane and zero average voltage in the non-energy transferring plane. Existing space-vector PWM (SVPWM) based LMTs show good ripple-current performances but few of them suffer from the disadvantages like, higher implementation complexity, multiple switching of a leg, and simultaneous switching of multiple legs over one carrier-cycle. A novel SVPWM based LMT is proposed in this paper where the above disadvantages are avoided and, therefore, can be implemented in a computationally efficient carrier based method. The proposed strategy achieves maximum possible voltage gain of LMTs and shows current-ripple performance better or close to carrier-comparison based best known existing techniques. The proposed technique is validated through simulation and experiments performed up to 3.5 kW with a laboratory-scale hardware prototype.


Index Terms-Asymmetrical six-phase machine, space-vector modulation, current-ripple, harmonic loss, linear modulation technique, multi-phase machine.

## I. Introduction

Asymmetrical six-phase machine (ASPM), one of the most popular multi-phase machines, has two sets of balanced threephase $(3 \phi)$ windings with a spatial angular difference of $30^{\circ}$ electrical, as shown in Fig. 1. Like other multi-phase machines, ASPM finds its applications in high-power drives, electric vehicles, and railway traction, safety-critical electric aircraft, ship-propulsion, etc. due to having advantages of reduced power rating of the per-phase power-electronic drive unit, better fault-tolerance, lesser susceptibility towards space and supply harmonics, [1]-[4].

Vector-space-decomposition based modeling analyses ASPM in three two-dimensional orthogonal subspaces, namely, $\alpha-\beta, z_{1}-z_{2}$ and $o_{1}-o_{2}$, [5]. Linear modulation techniques (LMTs) of inverter fed ASPM synthesize the desired reference voltage vector in the $\alpha-\beta$ plane, responsible for electromagnetic energy transfer, and zero


Fig. 1: Six-phase inverter fed ASPM
average voltages in non-energy transferring $z_{1}-z_{2}$ and $o_{1}-o_{2}$ planes to avoid unwanted current and associated copper loss. Existing LMTs of ASPM can be classified into two groups: a) Four-dimensional space-vector PWM (SVPWM) techniques, as proposed by [2], [6]-[8], b) Twoinverter (TINV) based techniques, [1], [9], [10], where two $3 \phi$ inverters are modulated with reference voltage vectors of the same magnitude but phase-shifted by $30^{\circ}$. Although SVPWM techniques show superior current-ripple performances compared to TINV techniques, most of them also suffer from major disadvantages, like higher implementation complexity, multiple switching in a carrier-cycle, simultaneous switching of multiple legs [2], [7], compared to TINV techniques.

In this paper, a novel SVPWM technique of two-level $6 \phi$ inverter fed ASPM is proposed, which doesn't have the problem of multiple switching and simultaneous switching over a carrier cycle. Hence, it can be implemented in a carriercomparison way like TINV techniques. The proposed method attains the maximum possible voltage gain for LMTs. The analytical closed-form expression of the RMS ripple-current over line-cycle is derived and given in the paper. The proposed


Fig. 2: Mapping of 16 neighbouring switching states of sector1
strategy is the best or close to the best-known techniques in terms of ripple performance while compared with existing carrier-based LMTs.

The organization of the paper is as follows: Modeling of ASPM and $6 \phi$ inverter is discussed in section-II; sectionIII briefly describes the existing linear PWM techniques of ASPM. The proposed technique and it's performance comparison with existing techniques are discussed in sectionIV and section-V, respectively. Finally the simulation and experimental results are discussed in section-VI and paper is concluded in section-VII.

## II. Modeling of Two-Level Six-phase Inverter Fed ASPM

Fig. 1 shows a two-level six-phase ( $6 \phi$ ) inverter fed ASPM. $a, b, c$ and $a^{\prime}, b^{\prime}, c^{\prime}$ are the terminals of two sets of three-phase $(3 \phi)$ windings, which are directly connected to the poles of $6 \phi$ inverter. These two sets of windings are spatially shifted by $30^{\circ}$ electrical and connected in star fashion with two isolated neutral points $o$ and $o^{\prime}$, respectively. The DC-bus voltage of the inverter is $V_{D C}$.

$$
\begin{align*}
& X_{i} \triangleq \frac{1}{\sqrt{3}}\left[\begin{array}{cccccc}
1 & -\frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\
1 & -\frac{1}{2} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 \\
0 & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{2} & -1 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1
\end{array}\right] \\
& \underbrace{}_{T} \\
& X_{i}=\left[\begin{array}{llllll}
x_{\alpha} & x_{\beta} & x_{z_{1}} & x_{z_{2}} & x_{o_{1}} & x_{o_{2}}
\end{array}\right]^{T}  \tag{1}\\
& X_{j}=\left[\begin{array}{llllll}
x_{a} & x_{b} & x_{c} & x_{a^{\prime}} & x_{b^{\prime}} & x_{c^{\prime}}
\end{array}\right]^{T}
\end{align*}
$$

As each leg of two-level inverter has 2 switching states, $6 \phi$ inverter of Fig. 1 has 64 switching states. These states are labelled by an ordered pair, like $\left(x, y^{\prime}\right)$, where $x$ and $y$ denote the switching states of $3 \phi$ inverters, Inverter-1 and Inverter-2 of Fig. 1, respectively. Table I shows these standard notations

TABLE I: Switching States of $3 \phi$ Inverter

| State | Label | State | Label |
| :---: | :---: | :---: | :---: |
| 000 | 0 | 011 | 4 |
| 100 | 1 | 001 | 5 |
| 110 | 2 | 101 | 6 |
| 010 | 3 | 111 | 7 |

of states of $3 \phi$ inverter. The entries under 'State' in Table I bear three binary numbers corresponding to switching states of first, second, and third legs of $3 \phi$ inverter, respectively. Here, ' 1 ' denotes top switch of a leg is 'on' and bottom switch is 'off'; ' 0 ' denotes the opposite. Therefore, $\left(1,6^{\prime}\right)$ state implies top switches of $a, a^{\prime}, c^{\prime}$ and bottom switches of $b, c, b^{\prime}$ are 'on'; other switches are 'off'.
A $6 \times 6$ transformation matrix, $T$, as given in (1), is widely used to model ASPM, [1]. $T$ transforms quantities from original six-dimensional domain to three two-dimensional orthogonal subspaces, $\alpha-\beta, z_{1}-z_{2}$, and $o_{1}-o_{2}$. The line-neutral voltages generated by 64 states of $6 \phi$ inverter can be mapped in $\alpha-\beta, z_{1}-z_{2}$, and $o_{1}-o_{2}$ with the help of matrix $T$. For example, line-neutral voltage vector in original domain generated by state $\left(1,6^{\prime}\right)$ is as follows.

$$
\left.V\right|_{\left(1,6^{\prime}\right)}=\frac{V_{D C}}{3}\left[\begin{array}{llllll}
2 & -1 & -1 & 1 & -2 & 1
\end{array}\right]^{T}
$$

The corresponding voltages in $\alpha-\beta, z_{1}-z_{2}$, and $o_{1}-o_{2}$ can be obtained by applying $T$ on $\left.V\right|_{\left(1,6^{\prime}\right)}$. The voltages in these three two-dimensional subspaces can be combined as real and imaginary parts of three space-vectors, as follows.

$$
\begin{gathered}
\left.\left(v_{\alpha}+j v_{\beta}\right)\right|_{\left(1,6^{\prime}\right)}=\frac{2 V_{D C} \cos 15^{\circ}}{\sqrt{3}} e^{-j 15^{\circ}} ; \\
\left.\left(v_{z_{1}}+j v_{z_{2}}\right)\right|_{\left(1,6^{\prime}\right)}=\frac{2 V_{D C} \cos 75^{\circ}}{\sqrt{3}} e^{-j 75^{\circ}} ;\left.\left(v_{o_{1}}+j v_{o_{2}}\right)\right|_{\left(1,6^{\prime}\right)}=0
\end{gathered}
$$

For a balanced ASPM with two isolated neutral points, as shown in Fig. 1, all 64 states generate zero voltages in $o_{1}-o_{2}$ plane and therefore, discussion of modulation in this plane is excluded in subsequent sections. The mapping of the lineneutral voltages, generated by these 64 states, in $\alpha-\beta$ and $z_{1}-z_{2}$ planes can be referred from [10]. Fig. 2 shows the mapping of those 16 states in $\alpha-\beta$ and $z_{1}-z_{2}$, which are required to discuss the existing and the proposed modulation techniques in one of the twenty four sectors of $\alpha-\beta$ plane. Conventionally, this is sector-1 and is shown in Fig. 2a. Sector numbers in Fig. 2a are surrounded by oval-shaped curves. The voltage vectors of state $\left(1,6^{\prime}\right)$ can be seen from Fig. 2. In Fig. 2, $z$ represents the zero state of $3 \phi$ inverter, i.e., $z \in$ $\{0,7\}$ according to Table I. Therefore, vectors involved with label $z$ or $z^{\prime}$ have redundant switching states. For example, both $\left(0,6^{\prime}\right)$ and $\left(7,6^{\prime}\right)$ are denoted by vector $\left(z, 6^{\prime}\right)$. So, the aforementioned 16 states result into 10 distinct vectors both in $\alpha-\beta$ and $z_{1}-z_{2}$ planes.

It can be seen that the non-zero voltage vectors are of three different lengths both in $\alpha-\beta$ and $z_{1}-z_{2}$ subspaces.

The switching states with large vector-length in $\alpha-\beta$ has tiny vector-length in $z_{1}-z_{2}$. For example, $\left(1,6^{\prime}\right)$ is such a switching state. Switching states with medium (e.g. state $\left(2,6^{\prime}\right)$ ) and small (e.g. state $\left(z, 6^{\prime}\right)$ ) vector-lengths in $\alpha-\beta$ keep the vector-lengths unchanged in $z_{1}-z_{2}$. These lengths of large (denoted by $L$ ), medium ( $M$ ), small ( $S$ ) and tiny ( $T$ ) vectors are given in (2).

$$
\begin{align*}
L=\frac{2 V_{D C} \cos \left(15^{\circ}\right)}{\sqrt{3}} ; & M=\frac{2 V_{D C} \cos \left(45^{\circ}\right)}{\sqrt{3}} ;  \tag{2}\\
S=\frac{V_{D C}}{\sqrt{3}} ; & T=\frac{2 V_{D C} \cos \left(75^{\circ}\right)}{\sqrt{3}}
\end{align*}
$$

Linear modulation techniques (LMTs) of ASPM synthesize the desired average voltage vector in $\alpha-\beta$, as this plane is associated with electromagnetic energy transfer, and zero average voltage vector in non-energy transferring $z_{1}-z_{2}$ plane in order to avoid unwanted average current and associated copper loss resulted due to the low impedance in this plane. A brief discussion of the existing LMTs and the proposed SVPWM based LMT and its performance comparison with existing techniques are discussed in subsequent sections.

## III. Existing Linear Modulation Techniques

Let, the reference voltage vector, $\overrightarrow{V_{r e f}} \triangleq \bar{v}_{\alpha}+j \bar{v}_{\beta}$, lies in sector-1, as shown in Fig. 2a. Here, bar represents the average over a carrier period. [5] shows that LMT of ASPM requires to apply atleast five distinct voltage vectors. [6] used four largest vectors adjacent to $\overrightarrow{V_{\text {ref }}}$, i.e., $\left(6,6^{\prime}\right),\left(1,6^{\prime}\right)$, $\left(1,1^{\prime}\right),\left(2,1^{\prime}\right)$, and multiple redundant zero vectors of the form $\left(z, z^{\prime}\right)$ in order to propose continuous and discontinuous 12 sector based space-vector based PWM (SVPWM) techniques, namely, C6 $\phi$ SVPWM12, D6 $\phi$ SVPWM12-A/B1/B2. These techniques have two major disadvantages, as follows.

1) turning on and turning off a semiconductor device more than one time within a carrier period.
2) simultaneous transitions of two devices pertaining to two different legs.
The first disadvantage causes increase in instantaneous switching frequency for the same average switching frequency, whereas the second one causes asymmetric PWM waveforms, [2], [7]. [7] then proposed 24 sector based LMTs, namely, C6 $\phi$ SVPWM24, D $6 \phi$ SVPWM24-B1/B2 by using three adjacent largest vectors $\left(\left(1,6^{\prime}\right),\left(1,1^{\prime}\right),\left(2,1^{\prime}\right)\right)$, one adjacent small vector $\left(\left(z, 1^{\prime}\right)\right)$ and zero vectors. Here, the first disadvantage is removed but the second disadvantage is still present in C6 $\phi$ SVPWM24 and D6 $\phi$ SVPWM24-B1. Another modified 24 sector based SVPWM technique, C $6 \phi$ SVPWM24-C, was proposed in [2] using three largest vectors, as SVPWM24 techniques, two small vectors $\left(\left(z, 1^{\prime}\right)\right.$ and $\left.\left(z, 6^{\prime}\right)\right)$ and zero vector. This doesn't have the above two disadvantages, but current THD performance is not as good as D6 $\phi$ SVPWM24 or D6 $\phi$ SVPWM12 techniques at higher voltage gain. Recently, [8] has proposed several 24 sector based PWM techniques among which a technique, named as $6 \phi \mathrm{SVM} 3$, shows the superior performance. $6 \phi$ SVM3 technique is a continuous


Fig. 3: Switching sequence of the proposed technique when $\overrightarrow{V_{\text {ref }}}$ is in sector-1

PWM technique (where all six legs experience switching transition over a carrier period) and it uses three nearest largest active vectors like C6 $\phi$ SVPWM24, one small vector, $\left(z, 6^{\prime}\right)$, and zero vector. Although [8] hasn't explicitly mentioned the sequence of $6 \phi \mathrm{SVM} 3$, the only possible sequence to construct a continuous technique using the above vectors is $\left(0,7^{\prime}\right)-\left(0,6^{\prime}\right)-\left(1,6^{\prime}\right)-\left(1,1^{\prime}\right)-\left(2,1^{\prime}\right)-\left(7,0^{\prime}\right)$. Note, the transition from $\left(2,1^{\prime}\right)$ to $\left(7,0^{\prime}\right)$ is accompanied by simultaneous switching of leg $c$ and leg $a^{\prime}$ (i.e., it has the second demerit).

Another set of existing LMTs of ASPM modulates Inverter1 and Inverter-2 with reference voltage vectors, $\frac{\overrightarrow{V_{\text {ref }}}}{2}$ and $\frac{V_{\text {ref }}}{2} e^{-j 30^{\circ}}$, respectively, using standard carrier-comparison based $3 \phi$ modulation techniques. [1] has shown that this kind of operation is mathematically equivalent to synthesis of $\overrightarrow{V_{r e f}}$ in $\alpha-\beta$ plane and zero average voltage vector in $z_{1}-z_{2}$ plane. This set of techniques are-a) Sine-triangle PWM (STPWM), [10], where zero common-mode voltage is injected between load-neutral and DC-bus mid-point; b) Double zerosequence injection PWM (DZIPWM or ZS3PWM), [9], [10], where two different common-mode voltages are injected for Inverter-1 and Inverter-2, which are equal to the half of the middle values of the corresponding $3 \phi$ modulating waves; c) ZS6PWM, [10], where a single common-mode signal is used for both Inverter-1 and Inverter-2 and this is equal to the half of negative summation of maximum and minimum of $6 \phi$ modulating waves. As these set of techniques are carrier-comparison based techniques, they are devoid of the aforementioned two disadvantages and easy to implement, but the current ripple performances are poor compared to SVPWM techniques as described earlier, [6], [10].

## IV. Proposed Linear Modulation Technique

The proposed technique uses $\left(0,7^{\prime}\right)-\left(0,6^{\prime}\right)-\left(1,6^{\prime}\right)-$ $\left(1,1^{\prime}\right)-\left(2,1^{\prime}\right)$ sequence to synthesize $\overrightarrow{V_{r e f}}$ in sector-1. These states are highlighted in blue in Fig. 2. Therefore, the proposed technique uses-i) three largest active vectors, adjacent to $\overrightarrow{V_{r e f}}$, ii) one small vector which is not adjacent to $\overrightarrow{V_{\text {ref }}}$ but away by $30^{\circ}$ to $45^{\circ}$ in $\alpha-\beta$ plane; iii) zero vector, as shown in Fig. 2a. The sequence of application of these vectors are-a) one zero state, $b$ ) the small vector, $c$ ) three largest vectors in the order they are closest to the small vector. The state corresponding
to the zero vector is chosen such that only one leg transits to move from both zero vector to the small vector and small vector to the next largest vector. With these given rules, one can select the switching state and design the sequence. For example, sequence of the states in sector- 24 would be $\left(0,0^{\prime}\right)-$ $\left(0,1^{\prime}\right)-\left(1,1^{\prime}\right)-\left(1,6^{\prime}\right)-\left(6,6^{\prime}\right)$. Fig. 3 shows the gating signals of the top switches of six legs of $6 \phi$ inverter corresponding to $\overrightarrow{V_{r e f}}$ in sector-1. It can be seen that none of the aforementioned two disadvantages is present in the proposed technique.

One should notice here that although the proposed technique and $6 \phi$ SVM3 use the same set of vectors, these two techniques are not same from current-ripple perspective due to following two reasons- a) distribution of zero-vector dwell time between the redundant zero-states, which impact the current ripple performance in $\alpha-\beta$ plane, are different; b) the numbers of switching transitions over a carrier-cycle are different of these two techniques. As the proposed technique results into turn-on and turn-off of four devices compared to six devices which is the case for $6 \phi$ SVM3, the carrier frequency of the proposed technique can be made $\frac{6}{4}=1.5$ times higher compared to $6 \phi$ SVM3 to keep the average switching frequency same. This impacts the current ripple performance, as will be seen later.

Let, $V_{i, \alpha \beta}$ and $V_{i, z_{1} z_{2}}$ are the voltage-vectors generated by $i^{\text {th }}$ switching state, used to synthesize $\overrightarrow{V_{r e f}}$, in $\alpha-\beta$ and $z_{1}-z_{2}$ subspaces, respectively, where $i \in\{1,2, . .5\}$. The duty-ratio, $D_{i}$, for which $i^{t h}$ state needs to be applied, can be determined after solving system of equations given in (3). For example, let $D_{1}, D_{2}, \ldots D_{5}$ are the duty-ratios for which states $\left(0,7^{\prime}\right),\left(0,6^{\prime}\right),\left(1,6^{\prime}\right),\left(1,1^{\prime}\right)$ and $\left(2,1^{\prime}\right)$ are applied while $\overrightarrow{V_{r e f}}$ in sector-1. In this case, $V_{5, \alpha \beta}=L e^{j 45^{\circ}} ; V_{5, z_{1} z_{2}}=T e^{-j 135^{\circ}}$. Similarly, one can find other vectors as well. After substituting $L, M, S$ and $T$ from (2), expressions of these five duty-ratios in sector-1 are determined and given in (4). Here, $m_{\alpha}=\frac{\bar{v}_{\alpha}}{V_{D C}}$ and $m_{\beta}=\frac{\bar{v}_{\beta}}{V_{D C}}$. Likewise, one can find the duty ratios in other sectors of $\alpha-\beta$ plane.

$$
\begin{gather*}
\sum_{i=1}^{5} V_{i, \alpha \beta} D_{i}=\overrightarrow{V_{r e f}} \triangleq \bar{v}_{\alpha}+j \bar{v}_{\beta} ; \\
\sum_{i=1}^{5} V_{i, z_{1} z_{2}} D_{i}=0 ; \quad \sum_{i=1}^{5} D_{i}=1  \tag{3}\\
D_{1}=1-m_{\alpha} ; \quad D_{3}=\frac{\sqrt{3}-1}{2}\left(m_{\alpha}-m_{\beta}\right) ; \quad D_{5}=m_{\beta} \\
D_{2}=\left(1-\frac{\sqrt{3}}{2}\right) m_{\alpha}-\frac{m_{\beta}}{2} ; \quad D_{4}=\frac{m_{\alpha}}{2}-\left(1-\frac{\sqrt{3}}{2}\right) m_{\beta} \tag{4}
\end{gather*}
$$

## V. Performance Comparison of the Proposed Technique With Existing Techniques

## A. Maximum Modulation Index:

In this section, the maximum attainable DC-bus utilization of the proposed technique will be determined. Let, the modulation index, $M_{I}$, be defined as the ratio of peak line-neutral voltage and DC-bus voltage, $V_{D C}$. Therefore, at sinusoidal
steady-state operation, line-neutral voltages of $a$ and $a^{\prime}$ phases are as follows, where $\theta=\omega t$, and $\omega$ is output angular frequency.

$$
\bar{v}_{a o}=M_{I} V_{D C} \cos \theta, \quad \bar{v}_{a^{\prime} o^{\prime}}=M_{I} V_{D C} \cos \left(\theta-\frac{\pi}{6}\right)
$$

The line-neutral voltages of $b / b^{\prime}$ and $c / c^{\prime}$ phases are of same magnitude but phase-shifted by $120^{\circ}$ and $240^{\circ}$, respectively, with respect to $a / a^{\prime}$ phase voltages. After applying transformation $T$ of (1) on these voltages, one gets $\overrightarrow{V_{\text {ref }}}=$ $\sqrt{3} M_{I} V_{D C} e^{j \theta}$, or $m_{\alpha}+j m_{\beta}=\sqrt{3} M_{I} e^{j \theta}$. Note, the tip of $\overrightarrow{V_{r e f}}$ lies within sector-1 for $0^{\circ} \leq \theta \leq 15^{\circ}$. Plugging these $m_{\alpha}, m_{\beta}$ in (4), the duty-ratios, $D_{1}$ to $D_{5}$, can be expressed as functions of $M_{I}$ and $\theta$, as follows.

$$
\begin{align*}
& D_{1}=1-\sqrt{3} M_{I} \cos \theta ; \quad D_{2}=\sqrt{6-3 \sqrt{3}} M_{I} \cos \left(\theta+\frac{5 \pi}{12}\right) \\
& D_{3}=\frac{3-\sqrt{3}}{\sqrt{2}} M_{I} \cos \left(\theta+\frac{\pi}{4}\right) ; \quad D_{5}=\sqrt{3} M_{I} \sin \theta \\
& D_{4}=\sqrt{6-3 \sqrt{3}} M_{I} \cos \left(\theta+\frac{\pi}{12}\right) \tag{5}
\end{align*}
$$

All of these duty-ratios should be greater than or equal to zero for $0^{\circ} \leq \theta \leq 15^{\circ}$ to make the implementation feasible. It can be seen from (5) that $D_{1}$ first violates the above inequality at $\theta=0^{\circ}$ when $M_{I}>\frac{1}{\sqrt{3}}$. That means, the maximum attainable $M_{I}$ of this technique is $\frac{1}{\sqrt{3}}=0.577$, which is same as the maximum $M_{I}$ achievable by all existing LMTs except STPWM and ZS6PWM. The maximum attainable $M_{I}$ for these two techniques are 0.5 and 0.517 , respectively.

## B. RMS Harmonic Current:

Total RMS ripple current over a carrier period of all six phases of ASPM can be derived after following the steps given in section VI-A-2 of [6]. It is shown that unlike $3 \phi$ machine, the current ripple performance of ASPM is not only function of $\overrightarrow{V_{r e f}}$, but also depends upon machine parameter, $k_{\sigma x y}$, which is the ratio of the inductances seen in $\alpha-\beta$ and $z_{1}-z_{2}$ planes, i.e., $k_{\sigma x y}=\frac{L_{\alpha \beta}}{L_{z_{1} z_{2}}}$. The expression of this total RMS ripple current over a carrier period, $\tilde{i}_{R M S}$, is given in (6), where $\tilde{i}_{R M S}$ is per-unitized with respect to base current $\frac{V_{D C}}{F_{s w} L_{\alpha \beta}}$. Here, $F_{s w}$ is average switching frequency. In (6), the factor $k_{f}$ is the ratio of average switching frequency and carrier frequency, $F_{s}$, i.e., $k_{f} \triangleq \frac{F_{s w}}{F_{s}}$. This factor is introduced to keep $F_{s w}$ same for all LMTs during comparison. $\tilde{\lambda}_{\alpha \beta}$ and $\tilde{\lambda}_{z_{1} z_{2}}$ are RMS ripple fluxes in $\alpha-\beta$ and $z_{1}-z_{2}$ planes, respectively.

$$
\begin{equation*}
\tilde{i}_{R M S}^{2}=k_{f}^{2}\left(\tilde{\lambda}_{\alpha \beta}^{2}\left(m_{\alpha}, m_{\beta}\right)+k_{\sigma x y}^{2} \tilde{\lambda}_{z_{1} z_{2}}^{2}\left(m_{\alpha}, m_{\beta}\right)\right) \tag{6}
\end{equation*}
$$

This sub-cycle RMS ripple current can be then integrated over a line-cycle for sinusoidal excitation of $\alpha-\beta$ with $\overrightarrow{V_{r e f}}=$ $\sqrt{3} M_{I} V_{D C} e^{j \theta}$, or $m_{\alpha}+j m_{\beta}=\sqrt{3} M_{I} e^{j \theta}$, as obtained before, to get the line-cycle ripple current RMS as function of $M_{I}$, as follows.


Fig. 4: Comparison of ripple current performances of seven carrier-based techniques with same average switching frequency

TABLE II: $k_{f}$ of different techniques

| Technique | $k_{f}$ |
| :---: | :---: |
| D6 $\phi$ SVPWM24-B2 | $\frac{2}{3}$ |
| STPWM, ZS6PWM, DZIPWM | 1 |
| C6 $\phi$ SVPWM24-C, $6 \phi$ SVM3 | 1 |
| The proposed technique | $\frac{2}{3}$ |

$$
\begin{array}{r}
\tilde{I}_{R M S}^{2}\left(M_{I}\right)=\frac{1}{6}\left(\frac{1}{2 \pi} \int_{0}^{2 \pi} \tilde{i}_{R M S}^{2}\left(M_{I}, \theta\right) d \theta\right)  \tag{7}\\
=k_{f}^{2}\left(\tilde{\Lambda}_{\alpha \beta}^{2}\left(M_{I}\right)+k_{\sigma x y}^{2} \tilde{\Lambda}_{z_{1} z_{2}}^{2}\left(M_{I}\right)\right)
\end{array}
$$

As the ripple RMS current seen by all six phases over a linecycle is same and $\tilde{i}_{R M S}$ denotes carrier cycle RMS current combining all six phases, factor $\frac{1}{6}$ is introduced in (7) to obtain the line-cycle ripple current RMS of each phase. $\tilde{I}_{R M S}$ is also per-unitized with respect to base current $\frac{V_{D C}}{F_{s w} L_{\alpha \beta}}$. The expressions of line-cycle RMS of ripple-fluxes in two planes, $\tilde{\Lambda}_{\alpha \beta}$ and $\tilde{\Lambda}_{z_{1} z_{2}}$, are given in (8) as functions of $M_{I}$, from which $\tilde{I}_{R M S}$ can be computed using (7).

$$
\begin{array}{r}
\tilde{\Lambda}_{\alpha \beta}^{2}\left(M_{I}\right)=0.1136 M_{I}^{4}-0.1353 M_{I}^{3}+0.0417 M_{I}^{2} \\
\tilde{\Lambda}_{z_{1} z_{2}}^{2}\left(M_{I}\right)=9 \times 10^{-4} M_{I}^{3} \tag{8}
\end{array}
$$

The machine parameter, $k_{\sigma x y}$, changes from full-pitch winding to short-pitched winding, [11], in case of doublelayer winding machine. The machine used in the experimental verification of [6] has $k_{\sigma x y}=1.58$ in full-pitched condition and $k_{\sigma x y}=11$ for $\frac{5}{6}$-th short-pitch winding. The fullpitched winding machine used in this paper for experimental verification also has $k_{\sigma x y}=1.69$. Based on these observations, $1.5 \leq k_{\sigma x y} \leq 10$ is considered to be the feasible range in this paper.

Fig. 4 compares $\tilde{I}_{R M S}$ vs $M_{I}$ of the proposed technique, as obtained from (7), and other existing techniques at three values of $k_{\sigma x y}$, .viz, 2, 6, 10. During this comparison, the average switching frequencies of all techniques are kept constant. In the proposed technique, four out of six legs experience one complete switching process (turn-on and turn-off) within one $T_{s}$, as can be seen from Fig. 3. Therefore, $k_{f}=\frac{2}{3}$ for this proposed technique. The techniques proposed in [6] are not considered in this comparison as they result into more than

TABLE III: Equivalent Circuit Parameters of $6 \phi$ IM

| Per phase stator and rotor resistances | $0.675 \Omega$ |
| :---: | :---: |
| Per phase magnetizing inductance | 0.186 H |
| High-frequency $\alpha-\beta$ plane inductance, $L_{\alpha \beta}$ | 2.97 mH |
| High-frequency $z_{1}-z_{2}$ plane inductance, $L_{z_{1} z_{2}}$ | 1.76 mH |
| Ratio of inductances in $\alpha-\beta$ and $z_{1}-z_{2}, \gamma$ | 1.69 |

TABLE IV: Operating Conditions of Experiments and Simulations

| DC-bus voltage $\left(V_{D C}\right)$ | 300 V |
| :---: | :---: |
| Output frequency at maximum $M_{I}\left(M_{I}=0.577\right)$ | 50 Hz |
| Average Switching Frequency $\left(F_{s w}\right)$ | 8.33 kHz |
| Output Power at maximum $M_{I}$ | 3.55 kW |

two transition of a leg within a carrier cycle, which can't be implemented in carrier-based ways and also increase the instantaneous switching frequency. The compared techniques are- 1) D6 $\phi$ SVPWM24-B2, the best technique of [7], 2) C6 $\phi$ SVPWM24-C, 3) STPWM, 4) DZIPWM, 5) ZS6PWM and 6) $6 \phi$ SVM3, the best technique of [8]. Table II tabulates $k_{f}$ values of these techniques.

It can be seen from Fig. 4 that the performance of C6 $\phi$ SVPWM24-C and $6 \phi$ SVM3 are almost similar for all values of $M_{I}$ in the entire feasible range of $k_{\sigma x y}$. For any given $k_{\sigma x y}$, $\mathrm{C} 6 \phi$ SVPWM24-C or $6 \phi$ SVM3 shows superior performance at lower $M_{I}$ whereas D6 $\phi$ SVPWM24-B2 and the proposed technique outperform at higher range of $M_{I}$. The superiority of the proposed technique compared to D6 $\phi$ SVPWM24-B2 (and other existing techniques at higher $M_{I}$ ) is clear from Fig. 4. It is seen that as $k_{\sigma x y}$ increases, the range of $M_{I}$, for which the proposed technique shows the best performance, also increases. For $k_{\sigma x y}>5$, the proposed technique is (or close to) the best technique for the entire range of $M_{I}$, as can be seen from Fig. 4 b and 4 c .

## VI. Simulation and Experimental Results

To validate the above analysis, experiments are performed on a a 2-pole, 5 kW asymmetrical $6 \phi$ induction machine (ASIM) and simulation is done in MATLAB-Simulink. Fig. 5j shows this ASIM which is coupled with a DC generator to load the ASIM. SKM75GB123D IGBT based $6 \phi$ inverter along with Zynq-7010 based controller, as shown in Fig. 5k, are used to verify the proposed technique experimentally. The


Fig. 5: Hardware set-up, experimental and simulated waveforms of proposed technique at $M_{I}=0.577$ and ripple-current comparison with existing techniques
equivalent circuit parameters of ASIM are obtained through standard tests as given in [12] and these parameters are given in Table III. Using these parameters, dynamic model of ASIM is implemented in Simulink to verify the proposed technique.

Experiments and simulations are performed at nine values of $M_{I}$ between 0.2 and 0.577 , .viz $M_{I} \in$ $\{0.2,0.25,0.3,0.35,0.4,0.45,0.5,0.55,0.577\}$, after keeping V/f constant. The operating conditions of these experiments or simulations are given in Table IV. As the proposed technique has $k_{f}=\frac{2}{3}$, the carrier frequency, $F_{s}$, is 12.5 kHz to get average switching frequency 8.33 kHz . Experimental and simulated two line-line voltages, $v_{a b}$ and $v_{a^{\prime} b^{\prime}}$, and four sinusoidal line currents, $i_{a}, i_{a^{\prime}}, i_{b}$, and $i_{b^{\prime}}$, are shown in Fig. 5b, 5d, 5a and 5 c , respectively, for $M_{I}=0.577$, output frequency of 50 Hz , and output power of 3.55 kW . The fundamental operation of ASPM can be verified from $30^{\circ}$ phase-shifted pairs, $\left\{v_{a b}\right.$, $\left.v_{a^{\prime} b^{\prime}}\right\},\left\{i_{a}, i_{a^{\prime}}\right\},\left\{i_{b}, i_{b^{\prime}}\right\}$ and $120^{\circ}$ phase-shifted pairs, $\left\{i_{a}\right.$,
$\left.i_{b}\right\},\left\{i_{a^{\prime}}, i_{b^{\prime}}\right\}$. Fig. 5e shows the six pole voltages, $v_{x N}$, while $\overrightarrow{V_{\text {ref }}}$ in sector-1 to show the switching sequence, where $x \in\left\{a, b, c, a^{\prime}, b^{\prime}, c^{\prime}\right\}$ and $N$ is DC-bus negative terminal. The switching sequence is same as that of Fig. 3.

The comparison of current ripple performance between the proposed and the existing techniques is first validated through simulations. Fig. 5f, 5 g , 5 h plot $\tilde{I}_{R M S}$ vs $M_{I}$ of the existing and the proposed techniques, as obtained from simulations, for $k_{\sigma x y}$ values of 2,6 , and 10 , respectively. As the performances of C6 $\phi$ SVPWM24-C and $6 \phi$ SVM3 are almost similar for all values of $k_{\sigma x y}$ and $M_{I}$, only the performance of C6 6 SVPWM24-C is compared. Both the analytical results of Fig. 4 and the simulated results reach to the same conclusion, as discussed before in section-V-B. The experimental validation of the ripple performance of the proposed technique is shown in Fig. 5i for the ASPM with $k_{\sigma x y}=1.69$. Close agreement between analytical, simulated
and experimental results verifies the proposed modulation strategy.

## VII. CONCLUSION

This paper proposes an SVPWM based modulation technique of ASPM, which synthesizes the reference voltage vector in the $\alpha-\beta$ plane and zero voltage vector in the $z_{1}-z_{2}$ plane. This technique uses three adjacent largest active vectors; one small vector, located $30^{\circ}$ to $45^{\circ}$ away from the reference vector in $\alpha-\beta$ plane; and zero vector. The proposed strategy doesn't suffer from the common disadvantages of SVPWM techniques, like, multiple switching of a leg, simultaneous switching of different legs, and, therefore, can be implemented in a simple carrier-comparison way. It attains the maximum possible modulation index, which is 0.577 . The analytical closed-form expression of line-cycle RMS of the ripple current is derived. It shows that the proposed method is (or close to) the best technique when compared with all existing carrierbased methods for a) the entire range of modulation indices when machine parameter $k_{\sigma x y}>5$, b) higher modulation indices when $k_{\sigma x y}<5$. The proposed strategy is validated through simulation and experiments with prototype hardware up to 3.5 kW power level.

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