

Transformer Winding Losses with Round Conductors for Duty-Cycle Regulated Square Waves

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Abstract—One of the limiting factor in the course of reducing the size of high frequency transformer is the temperature rise. The knowledge of transformer power loss is important to make an estimate of the temperature rise. The transformer winding loss due to a duty-cycle regulated square current waveform can be estimated by summing the copper loss due to each harmonic using Dowell's formula. The paper shows that a large number of harmonics have to be considered for the loss computation. It is shown that for solid-round wire conductors the losses decrease with increasing diameter and there is no optimal diameter for which the losses are minimum. This paper presents a closed form approximate expression for power loss for a particular range of diameters of round wire that does not require a large series summation. Results from this closed form expression are shown to have reasonable accuracy in comparison to the fourier series method and also validated using 2-D finite element method.

I. INTRODUCTION

Transformers designed with solid round-wire winding are widely used in resonant power converters, switch mode dc-dc converters, ac-ac converters [1]. High frequency transformers are compact in size and hence there is a need to correctly estimate the losses to limit the temperature rise. The losses in a transformer occur primarily in the winding and the core. The winding losses in high frequency transformer increase with frequency due to skin and proximity effects. The winding losses for foil conductors are computed in [2] by assuming a 1-D model of a transformer. The 2-D effects can increase the losses but occur only if the conductors are spaced significantly apart as shown in [3], [4]. The above mentioned applications allow the use of closely packed conductors and hence 1-D analysis can be used.

The winding loss analysis was extended to solid-round conductor by approximating the round conductors as foils [2], [5], [6]. But an orthogonality existed between skin and proximity effects which resulted in a more generalized analytical approach for loss computation of solid round conductors [7]. The method was further improved in [8]. At high frequencies both Dowell's [2] and Ferreira's [7] methods resulted in substantial errors of 60 % in eddy current loss computation for round conductors [9]. Analytical expressions using a look up table to compute the winding losses more accurately for round conductors were developed in [10]. If the layer porosity factor is high, Dowell's 1-D expression with some modifications can be used to compute the losses with reasonable accuracy [10], [11], [12]. Dowell's 1-D expression for round conductors was modified further in [13]. The modified winding loss expression was used to compute losses for a solid-round wire inductor for sinusoidal excitation [14].

Switching circuits in power electronics lead to non-sinusoidal current waveforms through transformer windings [15], [16]. The losses in the transformer windings will be more in case of non-sinusoidal currents because of harmonics. In [17], for a non-sinusoidal waveform, the winding loss at each harmonic frequency is evaluated and then summed up to give the total winding losses. In [17] winding losses are calculated for a unipolar rectangular waveform as encountered in forward converter topology. In [18], the winding losses are computed for non-sinusoidal currents in solid-round conductors. The winding losses in case of round conductors for non-sinusoidal waveforms will always be less as compared to the analytical losses computed using Dowell's approach [19], [20]. For a duty-cycle regulated square waveform the winding losses depend on a large number of harmonics [21]. In [21] an approximate expression to compute the power loss independent of harmonics for foil windings was shown. This paper computes the winding loss for duty-cycle regulated square waveform in a transformer with solid round-wire windings.

Section II shows the AC power loss expression for duty-cycle regulated square waveform and also shows that there is no optimal diameter of the conductor for which the AC power loss is minimum. A generalized relation between current and normalized diameter (independent of frequency) is also shown. Section III presents a range of normalized diameter which should be avoided. It also shows approximate power loss expressions for other ranges of normalized diameter. The approximate expression is verified using numerical computation. Section IV outlines a procedure to design the windings using round conductors for given specifications and hence compute the losses. Section V validates the proposed method with the analytical expression and FEM results. Finally, Section VI presents the conclusion.

II. WINDING POWER LOSS FOR ROUND WIRE WINDING

The ac to dc resistance ratio F_R for solid round-wire windings and sinusoidal current is [2], [14]

$$F_R = \Delta \left[\frac{\sinh(2\Delta) + \sin(2\Delta)}{\cosh(2\Delta) - \cos(2\Delta)} + \frac{2}{3}(p^2 - 1) \frac{\sinh(\Delta) - \sin(\Delta)}{\cosh(\Delta) + \cos(\Delta)} \right] \quad (1)$$

where Δ is,

$$\Delta = 0.75 \sqrt{\frac{\pi}{4}} \frac{d}{\delta} \sqrt{\eta} \quad (2)$$

The diameter of solid round-wire winding is d . δ is the skin depth at the fundamental frequency and η is d/l where, l is the distance between centers of adjacent round conductors as

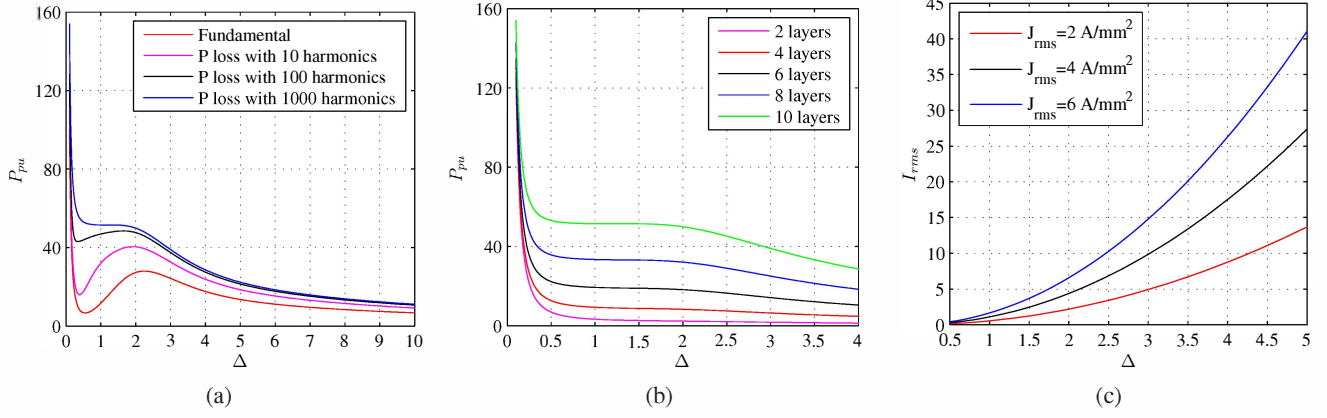


Fig. 3: (a) Plot of P_{pu} using (5) versus Δ for $p=10$ and $D=1$ for different numbers of harmonics (b) Plot of P_{pu} versus Δ for different layers for $D=1$ and using 1000 harmonics (c) Plot of I_{rms} versus Δ for different J_{rms} , for $f = 20kHz$ and $\eta = 0.9$

shown in Fig. 1. A bipolar duty-cycle regulated square current waveform is considered as shown in Fig. 2. This current can be represented by the following Fourier series:

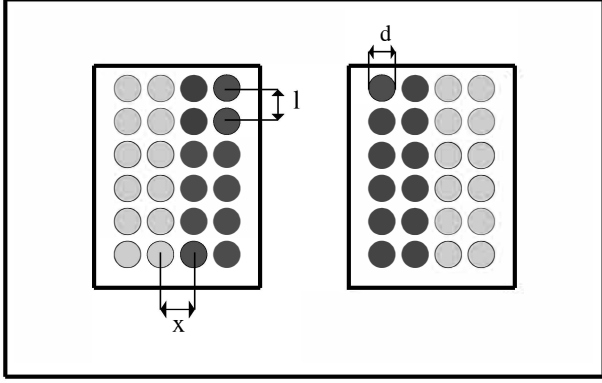


Fig. 1: Solid-round wire winding of a transformer.

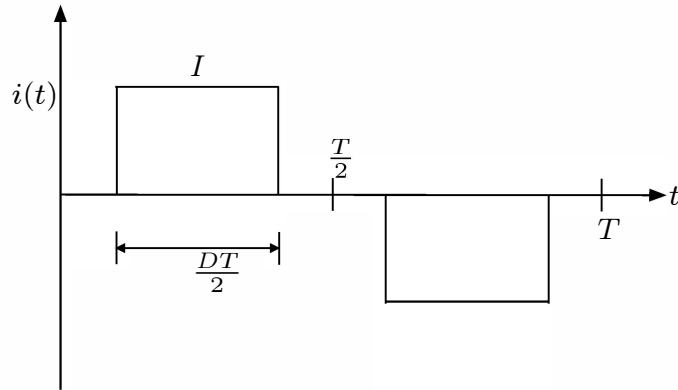


Fig. 2: Duty-cycle regulated current waveform.

$$i(t) = I_0 + \sum_{k=1}^{\infty} \sqrt{2} I_k \sin(k\omega t) \text{ where, } I_k = \frac{2\sqrt{2}I \sin(\frac{k\pi D}{2})}{k\pi} \quad (3)$$

and as the waveform is symmetric, $I_0 = 0$. The power loss expression for solid round-wire conductors for one winding can

be written as [2],

$$P = \sum_{k=1}^{\infty} \frac{8I^2 \sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}} \pi^2} \frac{\rho(MLT)N\eta}{\delta^2 \Delta} \sqrt{\frac{\pi}{4}} \times \left[\frac{\sinh(2\sqrt{k}\Delta) + \sin(2\sqrt{k}\Delta)}{\cosh(2\sqrt{k}\Delta) - \cos(2\sqrt{k}\Delta)} + \frac{2}{3}(p^2 - 1) \frac{\sinh(\sqrt{k}\Delta) - \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) + \cos(\sqrt{k}\Delta)} \right] \quad (4)$$

where, P is the AC power loss, D is the Duty ratio of square waveform, p is the number of layers, k is the harmonic number, ρ is the resistivity of copper, I is the peak value of square current, N_l is the number of turns per layer, N is the total number of turns ($= N_l p$) and MLT is the Mean Length of Turns. Here P_{base} is assumed to be, $P_{base} = \frac{I^2 \eta R_{dc}|_{d=\delta}}{\sqrt{\pi}}$ and the derivation is shown in APPENDIX. Hence,

$$\frac{P}{P_{base}} = P_{pu} = \sum_{k=1}^{\infty} \frac{\sin^2(\frac{k\pi D}{2})}{\Delta k^{\frac{3}{2}}} \times \left[\frac{\sinh(2\sqrt{k}\Delta) + \sin(2\sqrt{k}\Delta)}{\cosh(2\sqrt{k}\Delta) - \cos(2\sqrt{k}\Delta)} + \frac{2}{3}(p^2 - 1) \frac{\sinh(\sqrt{k}\Delta) - \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) + \cos(\sqrt{k}\Delta)} \right] \quad (5)$$

Fig. 3a, shows a plot of P_{pu} as the number of harmonics considered is increased. From Fig. 3a, it can be seen that for a sinusoidal waveform as shown in [14], there exists a valley diameter, hill diameter and a critical diameter whereas for duty-cycle regulated square waveform there is no such minimum. Fig. 3b shows a plot of P_{pu} for different layers by considering 1000 harmonics. The reduction in loss is not appreciable from $\Delta=0.5$ to $\Delta=4$. The implication of the results shown in Fig. 3b is that, any value of $\Delta > 0.5$ is acceptable if no other factors are considered. However, choosing $\Delta = 0.5$ so as to minimize the amount of copper is not possible in most cases because of

thermal considerations. The current density, can be written as,

$$J_{rms} = \frac{I_{rms}}{A} = \frac{I_{rms}}{\frac{\pi d^2}{4}} \quad (6)$$

Using, (2) and (6), yields,

$$\frac{I_{rms}}{J_{rms}} = \frac{2\Delta^2}{\sqrt[1.5]{\pi} f \mu \sigma \eta} \quad (7)$$

Simple thermal considerations will yield a maximum allowable J_{rms} for a given core size. Thus for a given J_{rms} and I_{rms} in (7), there will be an associated value of Δ which may be greater than 0.5. Examples of I_{rms} versus Δ are shown in Fig. 3c for different values of J_{rms} .

III. APPROXIMATE POWER LOSS EXPRESSION FOR DIFFERENT RANGES OF Δ .

As shown in the previous section, a large number of harmonics have to be considered to compute the AC power loss accurately for a duty-cycle regulated square waveform. As shown in Fig. 3b there are three ranges of Δ for which P_{pu} has distinct behavior. This allows for considerable simplification in the estimation of P_{pu} .

A. Case A : $\Delta < 0.5$

Fig. 3b, shows a plot of P_{pu} for different layers. From Fig. 3b, it can be seen that the P_{pu} curve for all layers is very steep for $\Delta < 0.5$. For a fixed current I , as J is increased, Δ will reduce, resulting in less copper. Hence, operating at a value of J allowed by thermal consideration should be appropriate. But in this case, even if the current density value is permissible for $\Delta < 0.5$, the use of that diameter should be avoided to prevent significant losses. The designer can use a bigger diameter to reduce the losses drastically at the cost of increased copper or use either of litz wires or foil conductors. Solving (7), by considering that the maximum value of current density is $6A/mm^2$, the limiting value of $\Delta = 0.5$ and $\eta = 0.9$:

$$I_{rms} = 8.213/f \quad (8)$$

where, f is in kHz. As an example, for $f = 10kHz$, $I_{rms} > 0.82A$ for a current density of $J_{rms} = 6A/mm^2$. If for a particular application $I_{rms} < 0.82A$, a smaller value of current density should be chosen to avoid the value of $\Delta < 0.5$.

B. Case B: $0.5 < \Delta < 2.5$

The power loss computation for $0.5 < \Delta < 2.5$ depends on the harmonics and hence is computationally difficult. For $\Delta > 2.5$, both underlined functions in (9) become 1 as shown in [22] and hence, the power loss expression can be split as a finite sum and an infinite sum as shown in (9) below,

$$P_{pu} = \sum_{k=1}^{N_{\Delta}} \frac{\sin^2\left(\frac{k\pi D}{2}\right)}{k^{\frac{3}{2}} \Delta} \times \left[\frac{\sinh(2\sqrt{k}\Delta) + \sin(2\sqrt{k}\Delta)}{\cosh(2\sqrt{k}\Delta) - \cos(2\sqrt{k}\Delta)} + \frac{2}{3}(p^2 - 1) \frac{\sinh(\sqrt{k}\Delta) - \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) + \cos(\sqrt{k}\Delta)} \right] + \left[\left(\frac{2p^2 + 1}{3\Delta} \right) \sum_{k=N_{\Delta}+1}^{\infty} \frac{\sin^2\left(\frac{k\pi D}{2}\right)}{k^{\frac{3}{2}}} \right] \quad (9)$$

where, N_{Δ} is the harmonic number beyond which the underlined functions become independent of Δ . N_{Δ} can be found out using, $N_{\Delta} = \left(\frac{2.5}{\Delta} \right)^2$. The maximum value of N_{Δ} is 25, corresponding to $\Delta = 0.5$. The infinite sum is a convergent function and hence the power loss can be written as,

$$P_{pu} = \sum_{k=1}^{N_{\Delta}} \frac{\sin^2\left(\frac{k\pi D}{2}\right)}{k^{\frac{3}{2}} \Delta} \times \left[\frac{\sinh(2\sqrt{k}\Delta) + \sin(2\sqrt{k}\Delta)}{\cosh(2\sqrt{k}\Delta) - \cos(2\sqrt{k}\Delta)} + \frac{2}{3}(p^2 - 1) \frac{\sinh(\sqrt{k}\Delta) - \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) + \cos(\sqrt{k}\Delta)} - \frac{2p^2 + 1}{3} \right] + \left[\left(\frac{2p^2 + 1}{3\Delta} \right) \times S \right] \quad (10)$$

where,

$$S = \sum_{k=1}^{\infty} \frac{\sin^2\left(\frac{k\pi D}{2}\right)}{k^{\frac{3}{2}}} \quad (11)$$

TABLE I: Infinite series summation value for different D

Duty cycle	$D = 0.25$	$D = 0.5$	$D = 0.6$	$D = 0.75$	$D = 0.8$	$D = 1$
S	1.0785	1.4413	1.5336	1.6293	1.6508	1.6886

The value of the infinite sum depends on duty-ratio D and its value is known as shown in Table I. Hence only a series summation of 25 harmonics is to be considered instead of an infinite series to compute the AC losses.

C. Case C: $\Delta > 2.5$

As shown in the previous section, the P_{pu} for $\Delta > 2.5$ is simplified as under,

$$P_{pu} = \left[\left(\frac{2p^2 + 1}{3\Delta} \right) \sum_{k=1}^{\infty} \frac{\sin^2\left(\frac{k\pi D}{2}\right)}{k^{\frac{3}{2}}} \right] \quad (12)$$

Solving (7), by considering that the maximum value of J is $6A/mm^2$, the limiting value of $\Delta = 2.5$ and $\eta = 0.9$:

$$I_{rms} = 205/f \quad (13)$$

where, f is in kHz. If the rms current value is greater than the specified value, then $\Delta > 2.5$. As an example, if $I_{rms} > 4.1A$ then $\Delta > 2.5$ for $f = 50kHz$, $J_{rms} = 6A/mm^2$ or less. For $\Delta > 2.5$, the computation of AC losses is extremely simple and the P_{pu} , can be expressed as under,

$$P_{pu} = S \times \left(\frac{2p^2 + 1}{3\Delta} \right) \quad (14)$$

S can be obtained from Table I. Fig. 5b shows the percentage error in power loss computation by considering 2000 harmonics using (5) and, using (14) which is independent of harmonics, for a range of Δ from 2.5 – 10 and for 1 – 10 layers.

IV. PROCEDURE TO COMPUTE LOSSES IN A TRANSFORMER WITH ROUND CONDUCTORS

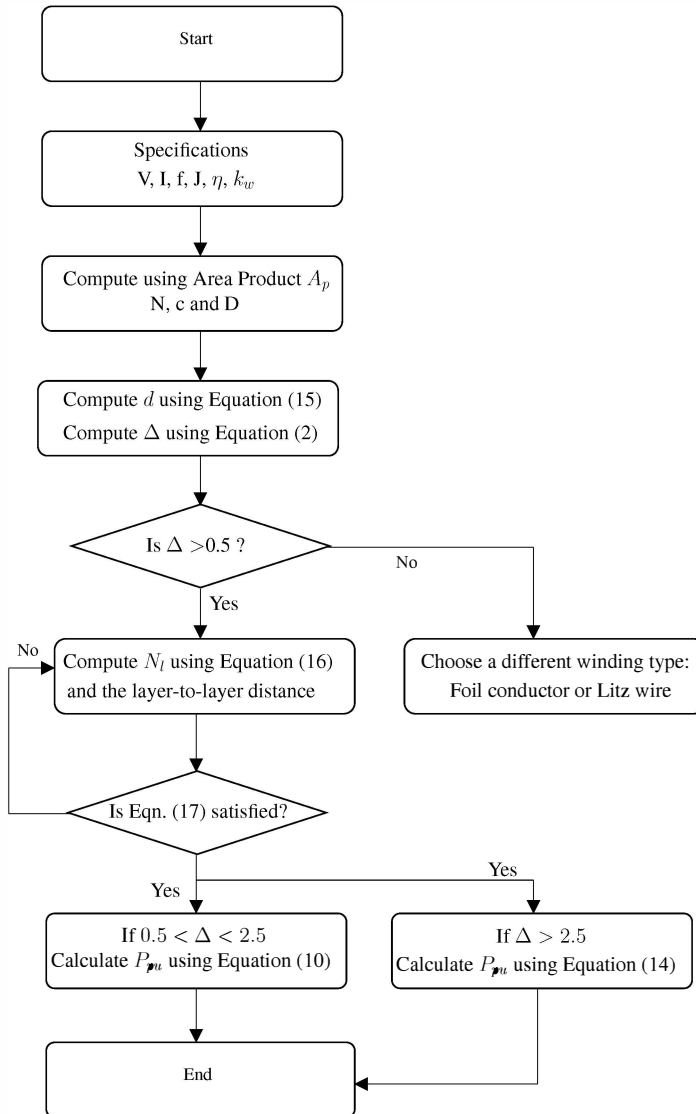


Fig. 4: A Step-by step procedure to compute winding losses in a high frequency transformer with round conductors.

For round conductors, the conductor area depends only on diameter, d . Fig. 4 shows a detailed design procedure to compute the losses in a high frequency transformer designed with solid-round conductors due to a duty-cycle regulated square waveform.

- 1 From the given specifications: Voltage, V , Current, I , frequency, f , Current Density, J , and window fill factor, k_w the core dimensions and the Number of turns, N can be computed using area product method.
- 2 For solid-round wire, the current, I and current density, J are fixed for a specific design, hence the diameter of round conductor is fixed, which can be computed using, (15). Compute Δ using (2).

$$d = \sqrt{\frac{4J}{\pi I}} \quad (15)$$

$$N_l = \frac{2\eta_1 c}{\sqrt{\pi} d} \quad (16)$$

$$\frac{2\sqrt{\pi} d p}{\eta_2} < D \quad (17)$$

where, η_2 is $\frac{d}{x}$, where, x is the distance between centers of two consecutive layers. For a 1:1 transformer the distance between primary and secondary layer can also be considered to be x , but for a transformer with a different transformation ratio the above simplification does not hold. For a different transformation ratio, the insulation distance between the primary and secondary layer will be more as compared to layer-to-layer insulation. η_1 is the layer porosity factor as defined in [14].

- 3 If $\Delta < 0.5$, then as explained in Case-A, it is better to use Litz wire or Foil conductors.
- 3 If not, then N_l can be determined from window height, c using (16)
- 4 From the number of layers, N_l compute p . For solid-round conductors the interleaving between primary and secondary windings will lead to lower losses. But if the transformation ratio is different, then the distance between primary and secondary winding will be more than x and hence it is important to check if (17) is satisfied.
 - a) If yes, then go to step 5.
 - b) If the inequality in (17) is not satisfied, then two or more primary layers have to be stacked together, resulting in increased losses.
- 5 If $0.5 < \Delta < 2.5$, compute the power loss using (10) but if $\Delta > 2.5$, use (14).

V. VALIDATION OF RESULTS

A 2-D Finite element analysis is done to verify the power loss for square waveform of duty ratio $D = 1$, with a rise time of 0.001% using ANSYS MAXWELL 16.0. Simulations are done at 10kHz, for 2 and 4 layers and for per unit current excitation. Δ depends on the current and the frequency. The validation is done for a range of Δ from (0.4–4.0). This allows to validate the results for a wide range of current (0.05–100)A and frequency (5–100)kHz and hence a wide range of design specifications. For different design specifications the core sizes will be different. Hence, the window and core sizes are scaled according to Δ to fit the required number of turns/layer. 60 turns for primary and secondary each and 4 values of Δ ranging from 0.4 – 4 are considered. Fig. 5a shows the current density distribution on one side of window of an EE-core for both primary and secondary round conductors containing 4 layers

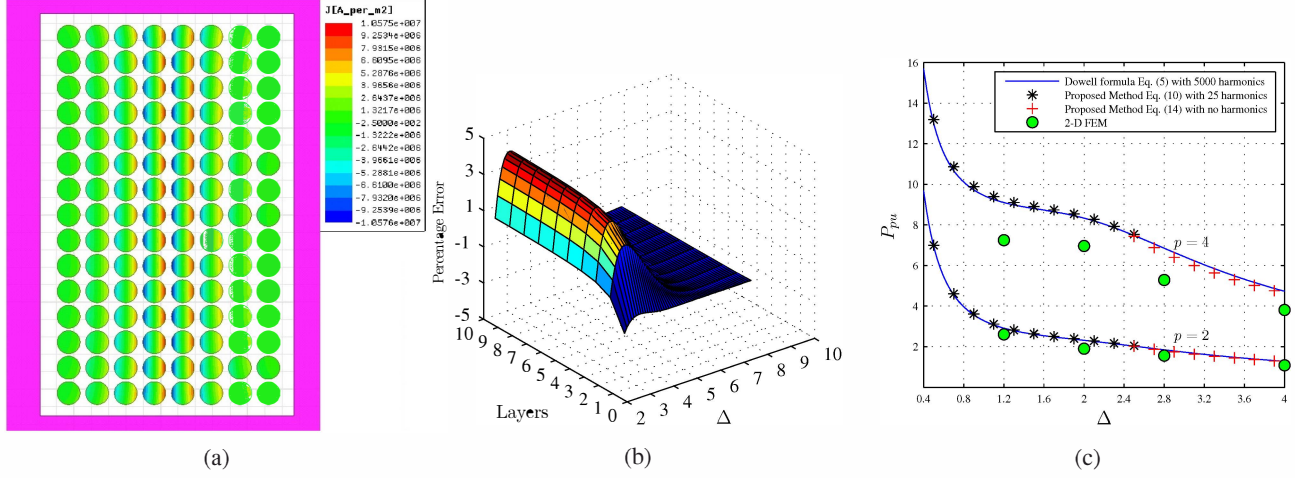


Fig. 5: Validation of results : (a) Current density distribution by 2-D FEM for 4 layers (b) Plot of percentage error in P_{pu} for 1-10 layers and $\Delta > 2.5$ by solving (5) considering 2000 harmonics and solving (14) (c) Plot of P_{pu} comparison between analytical and 2-D FEM simulation for 2 and 4 layers for $D = 1$ square waveform

TABLE II: Specifications

Parameter	Power	Voltage	Frequency, f	Current Density, J	B_{max}	k_w
Design I	5 kW	500 V	50 kHz	4 A/mm ²	0.25 T	0.4
Design II	1 kW	200 V	20 kHz	5 A/mm ²	0.3 T	0.4

each for a unit excitation of current. $\eta = 0.9$ is assumed for all simulations.

P_{pu} is computed analytically using (5) considering 5000 harmonics. P_{pu} is also computed using (10) considering 25 harmonics for $0.5 < \Delta < 2.5$ and using (14) for $\Delta > 2.5$ for 2 and 4 layers. The AC losses are also computed using 2-D FEM and converted into per unit for a square waveform excitation of 1.11072A corresponding to 1A rms fundamental. Fig. 5c show a comparison of P_{pu} computed using different methods for 2 and 4 layers respectively. As shown in Fig. 5c, the approximate expressions are quite close to the analytical values with negligible errors.

Fig. 5c validates the proposed power loss formula for a general case.

The design flowchart as given in Fig. 4 is validated for two different specifications and the winding losses are computed.

A. Design I and Design II

Fig. 6a and 6b show the 2-D model of a transformer winding designed using the round conductors. The specifications for Design I and Design II are outlined in Table II. Table III shows the core chosen for the particular design using the area product method. The number of turns in each layer and the number of layers for both primary and secondary winding are determined using Fig. 4. For Design I $\Delta=4.898$ and as $\Delta > 2.5$, P_{pu} can be computed using (14) which is 125.4 W. The losses can also be computed using (5), which gives 125 W close to the proposed method. The losses computed using 2-D FEM are 99.8 W.

For Design II as $\Delta=1.9488$, P_{pu} is computed using (10) which is 33.2 W. The losses can also be computed using (5), which gives 33.1 W and using 2-D FEM gives 27.7 W.

The losses computed using the proposed method match closely with the analytical results but are more when compared

TABLE III: Computed Parameters for Loss Estimation

Parameter	Core	No. of Turns, N	Δ	MLT
Design I	OP45528EC	28	4.898	0.1376 m
Design II	OP44721EC	36	1.9488	0.125 m

to 2-D FEM as shown in [20] and further validated by [19]. But due to the consideration of large number of harmonics the losses computed using the proposed method are much closer to 2-D FEM results.

VI. CONCLUSION

In this paper, an approximate expression for computing AC power losses of solid round-wire transformer windings for duty-cycle regulated square waveform has been presented. The AC losses keep decreasing with increasing diameter. Hence, there is no optimum diameter of solid round-wire where the AC losses are minimum. It has been shown that for a given current density, depending on the frequency, there is a maximum current value below which the AC losses will be higher. For currents below the maximum, it is advisable to operate at lower current densities to achieve reduced losses at the cost of increased copper. To compute power loss for currents above the maximum current value, the infinite sum is represented as two finite summations. In this paper, it is also shown that for a given current density, depending on the frequency, there is a minimum current value above which the AC power loss can be represented by a simplified expression independent of harmonics. The accuracy of the expression is verified using numerical computation as well as with 2-D FEM.

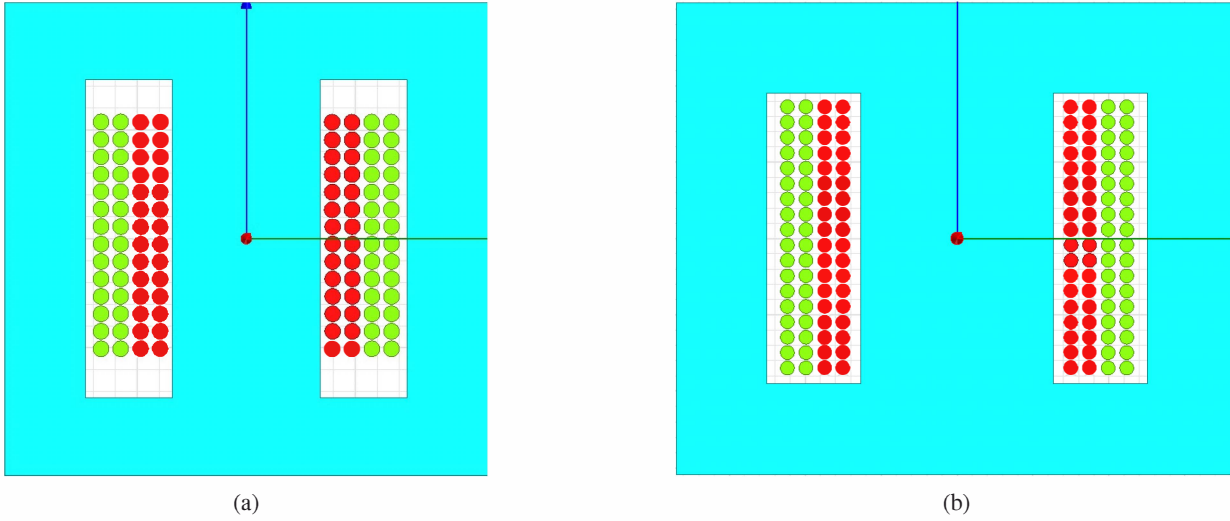


Fig. 6: 2-D Transformer model with round conductors: (red)-primary winding, (green)-secondary winding: (a) Design I (c) Design II

APPENDIX

The dc resistance of a transformer containing N multiple turns at a diameter equal to the skin depth of the conductor is given in (A.1). Now P_{base} given by (A.2) can be written in terms of $R_{dc}|_{d=\delta}$ as shown in (A.3).

$$R_{dc}|_{d=\delta} = \frac{N\rho(MLT)}{\frac{\pi}{4}\delta^2} \quad (\text{A.1})$$

$$P_{base} = \frac{8I^2}{\pi^2} \frac{N\rho(MLT)\eta}{\delta^2} \sqrt{\frac{\pi}{4}} \quad (\text{A.2})$$

$$P_{base} = \frac{I^2\eta R_{dc}|_{d=\delta}}{\sqrt{\pi}} \quad (\text{A.3})$$

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