Determination of the Optimal Thickness for a Multi-Layer Transformer Winding

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***Abstract*—High frequency transformers (HFT) are needed along with power electronic converters to replace line frequency transformers in high power systems to increase power density. For the design of HFT, it is important to accurately estimate copper losses due to a duty-cycle modulated current waveform. The design also requires determination of the optimal thickness of winding layers, leading to a minimum AC power loss. This paper shows that the Fourier- series method for the loss computation requires the consideration of a large number of harmonics, leading to considerable computational time in the determination of the optimal thickness. A closed form approximate expression for the power loss is presented in this paper that obviates any need for a large series summation, resulting in a relatively simple computation of optimal thickness. Results are validated through numerical computations.**

d	Thickness of the foil winding.
δ	Skin depth at the fundamental frequency.
Δ	d/δ.
P	AC power losses.
D	Duty ratio.
p	Number of layers.
k	Harmonic number.
a	Height of a conductor.
h	Height of winding.
ρ	Resistivity of copper.
I	Peak value of current.
N_l	The number of turns per layer.
N	Total number of turns ($= N_l p$).
Δ_{opt}	Optimal value of Δ.
MLT	Mean Length of Turns.
η	$N_l a/h$.

I. Introduction

In the recent literature there is a lot of emphasis on replacing line frequency high power transformers with their high frequency counterpart along with associated power electronics in order to increase power density. High frequency transformers are much smaller in size in comparison to its low frequency 60 Hz counterpart. Due to smaller effective area to dissipate heat, it is important to accurately estimate and minimize the winding losses. The copper losses in the transformer increase with frequency because of skin and proximity effects. In [1] winding losses are computed by considering a 1-D model of a transformer with foil windings. In [2] winding losses are calculated for a unipolar rectangular waveform as encountered in forward converter topology. In [1] and [2], for a non-sinusoidal waveform, the winding loss at each harmonic frequency is evaluated and then summed up to give the total winding losses. In [3] losses are calculated in rectifier transformers by approximating the functions given in [1]. [4] develops this idea of approximating the function to calculate the optimal thickness of a conductor for various different current waveforms. The given formula in [4] to determine the optimal thickness was used in [5] to compare the single layer and multi-layer windings. The optimal thickness formula given in [4] and [5], is not very accurate for three layers or less. Also, as the formula depends on differentiation of the current waveform, it cannot be used in case of duty cycle regulated square waveforms with negligible rise times. Typically, switching times of high power IGBTs are in order of hundreds of nano-seconds and switching frequency is in tens of kilohertz.

This paper considers foil windings, as shown in Fig. 1, but the analysis can also be applied to round conductors as shown in [6]. For a sinusoidal current waveform the thickness of the winding conductor for minimal power loss has been already computed. But for a duty-cycle regulated square waveform a large number of harmonics have to be considered as shown in the next section. In this paper, it is shown that for large number of layers, large number of harmonics have to be considered to accurately predict AC power loss. This also implies difficulty in the computation of optimal winding thickness by considering AC power loss as a sum of large number of harmonics. An approximate formula for AC power loss is derived in this paper that does not involve any series summation with large number of terms and leads to comparatively easy determination of the optimal winding thickness for a given number of layers and duty cycle D.

II. Variation of AC Power loss with harmonics

A bipolar duty cycle modulated square current waveform is considered as shown in Fig. 2. This current can be represented by the following Fourier series:

$$i(t) = I_0 + \sum_{k=1}^{\infty} \sqrt{2} I_k \sin(k\omega t) \tag{1}$$

where,

$$I_k = \frac{2\sqrt{2} I \sin(\frac{k\pi D}{2})}{k\pi} \tag{2}$$

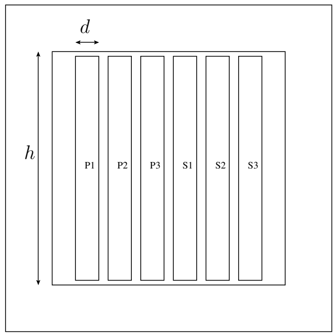

Fig. 1. Multi-layer foil winding of a transformer.

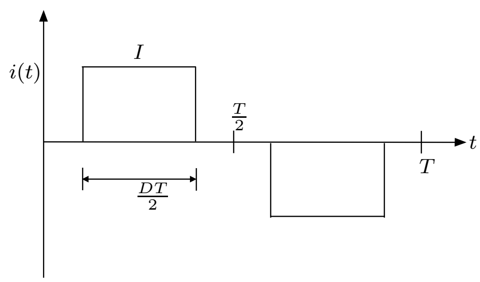

Fig. 2. Duty regulated current waveform.

and as the waveform is symmetric,

$$I_0 = 0 \tag{3}$$

The AC power loss expression can be written as [1],

$$\begin{aligned} P \;=\; & \sum_{k=1}^{\infty} \frac{8I^2 \sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}\pi^2} \frac{\rho(MLT)N^2}{h\delta p\eta} \\ & \times \left[\frac{\sinh(2\sqrt{k}\Delta) + \sin(2\sqrt{k}\Delta)}{\cosh(2\sqrt{k}\Delta) - \cos(2\sqrt{k}\Delta)} \right. \\ & \left. + \frac{2}{3}(p^2-1) \frac{\sinh(\sqrt{k}\Delta) - \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) + \cos(\sqrt{k}\Delta)} \right] \end{aligned} \tag{4}$$

Here, it is assumed that,

$$P_{base} = \frac{8I^2}{\pi^2} R_{dc}|_{d=\delta} \tag{5}$$

where,

$$R_{dc}|_{d=\delta} = \frac{N^2 \rho(MLT)}{h\delta p\eta} \tag{6}$$

is explained in the APPENDIX. Hence, the per unit power loss P_{pu},

$$\begin{aligned} \frac{P}{P_{base}} \;=\; & P_{pu} = \sum_{k=1}^{\infty} \frac{\sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}} \\ & \times \left[\frac{\sinh(2\sqrt{k}\Delta) + \sin(2\sqrt{k}\Delta)}{\cosh(2\sqrt{k}\Delta) - \cos(2\sqrt{k}\Delta)} \right. \\ & \left. + \frac{2}{3}(p^2-1) \frac{\sinh(\sqrt{k}\Delta) - \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) + \cos(\sqrt{k}\Delta)} \right] \end{aligned} \tag{7}$$

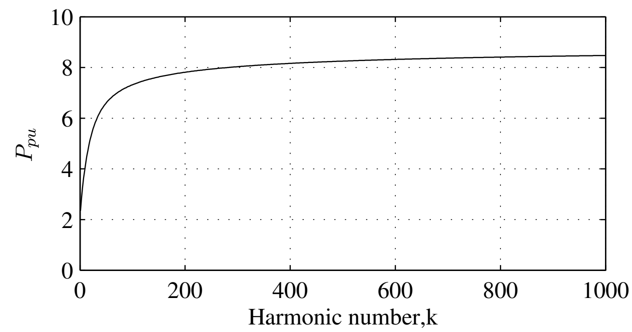

Fig. 3. Plot of P_{pu} using (7) versus k at $\Delta = 0.5$, for p=5 and D=1.

As seen from (7), each harmonic component of P_{pu} falls as a function of $k^{\frac{3}{2}}$. For a duty cycle modulated square current waveform, large number of harmonics have to be considered, as $k^{\frac{3}{2}}$ is a slowly decaying function using (7). Fig. 3, shows a plot of P_{pu} as a function of k at a given $\Delta = 0.5$, for 5 layers and $D = 1$. Ideally, infinite number of harmonics have to be considered but in practice only a finite number of harmonics can be used to compute P_{pu} approximately. But Fig. 3, shows that large number of harmonics have to be considered to avoid significant errors. Hence, it is computationally difficult to calculate the power loss using Fourier analysis method.

By using trigonometric identities (7) can be written as,

$$P_{pu} = \sum_{k=1}^{\infty} \frac{\sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}} \left[\frac{G_1}{2} + \frac{(4p^2-1)}{6} G_2 \right] \tag{8}$$

where G_1 and G_2 are,

$$G_1 = \frac{\sinh(\sqrt{k}\Delta) + \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) - \cos(\sqrt{k}\Delta)} \tag{9}$$

$$G_2 = \frac{\sinh(\sqrt{k}\Delta) - \sin(\sqrt{k}\Delta)}{\cosh(\sqrt{k}\Delta) + \cos(\sqrt{k}\Delta)} \tag{10}$$

The asymptotic values of both the functions in (9) and (10) approach 1 for $\sqrt{k}\Delta > 2.5$, as given in [4]. Hence, for each Δ there is a particular harmonic number beyond which the product $\sqrt{k}\Delta > 2.5$ and the harmonic component of the power loss becomes independent of Δ. For $\sqrt{k}\Delta > 2.5$, both (9) and (10) become 1 and P_k, the k^{th} component of P_{pu} is,

$$P_k = \frac{\sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}} \left(\frac{2p^2+1}{3} \right) \tag{11}$$

For smaller Δ as shown in [4], P_k can be written as,

$$P_k = \frac{\sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}} \left[\frac{1}{\sqrt{k}\Delta} + \left(\frac{5p^2-1}{45} \right) k^{3/2} \Delta^3 \right] \tag{12}$$

Now, (7) which is an infinite series sum depending on Δ, can be split into a finite sum that depends on Δ and an infinite sum independent of Δ.

$$P_{pu} = \left[\frac{1}{\Delta} \sum_{k=1}^{N_\Delta} \frac{\sin^2(\frac{k\pi D}{2})}{k^2} + \left(\frac{5p^2-1}{45} \right) \Delta^3 \right. \\ \times \sum_{k=1}^{N_\Delta} \sin^2\left(\frac{k\pi D}{2}\right) + \left(\frac{2p^2+1}{3} \right) \\ \left. \times \sum_{k=N_\Delta+1}^{\infty} \frac{\sin^2\left(\frac{k\pi D}{2}\right)}{k^{\frac{3}{2}}} \right] \quad (13)$$

where, N_Δ is the harmonic number beyond which the harmonic components of P_{pu} does not depend on Δ. Now, the third term in (13) can be written as,

$$\sum_{k=N_\Delta+1}^{\infty} \frac{\sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}} = \sum_{k=1}^{\infty} \frac{\sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}} - \sum_{k=1}^{N_\Delta} \frac{\sin^2(\frac{k\pi D}{2})}{k^{\frac{3}{2}}} \quad (14)$$

The three finite summations can be represented as a function of N_Δ, for a given duty ratio D, by curve fitting formula. The curve fitting formula is of the form $aN_\Delta^b + c$. By using the relation, $N_\Delta = (\frac{2.5}{\Delta})^2$ the power loss P_{pu} can be represented in terms of Δ. Hence, (13) takes the form,

$$P_{pu} = \frac{1}{\Delta}\left[k_1\left(\frac{2.5}{\Delta}\right)^{k_2} + k_3\right] + \left(\frac{5p^2-1}{45}\right) \\ \left(3.125\Delta + k_4\Delta^3\right) + \left(\frac{2p^2+1}{3}\right) \\ \times \left[k_5 - \left(k_6\left(\frac{2.5}{\Delta}\right)^{2k_7} + k_8\right)\right] \quad (15)$$

The value of k_5 is an infinite series sum which is known. The constants k_1 to k_8 in (15) are functions of duty cycle D. These constants are given in Table I for different values of D. For a given current waveform (with a specific value of D), (15) can be used to estimate the power loss for a given thickness Δ and number of layers, p.

TABLE I
VALUE OF CONSTANTS FOR SPECIFIC DUTY RATIOS

Coefficients	$D = 0.25$	$D = 0.5$	$D = 0.6$	$D = 0.75$	$D = 0.8$	$D = 1$
k_1	-0.4153	-0.4103	-0.3173	-0.3165	-0.3029	-0.4505
k_2	-0.9201	-0.9677	-0.9111	-0.821	-0.7975	-0.979
k_3	0.5402	0.9252	1.037	1.158	1.186	1.234
k_4	0.2426	0.25	0.2507	0.2512	0.2513	0.2515
k_5	1.0785	1.4413	1.5336	1.6293	1.6508	1.6886
k_6	-0.9608	-0.9089	-0.877	-0.8393	-0.8304	-0.8145
k_7	-0.4904	-0.4706	-0.4539	-0.4325	-0.4272	-0.4175
k_8	1.079	1.445	1.541	1.642	1.665	1.705

III. Computation of Δ_{opt}

There have been several ways proposed to calculate Δ_{opt}, [4] [7]. In [4], Δ_{opt} is represented in terms of the rms of differentiated current waveform, which is a closed form formula but it cannot be used for duty cycle modulated square waveforms with negligible risetimes. In [4], the entire series was approximated by (12). This approximation is not particularly true for low number of layers.

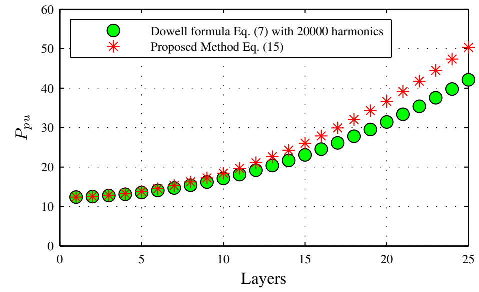

Fig. 4. Comparison of different methods for computation of P_{pu}.

To compute Δ_{opt} using (7), N_Δ should be known. For smaller layers the value of Δ_{opt} is close to 1, hence N_Δ is small. Therefore, for smaller layers, only few harmonics are required for computation of Δ_{opt}. Whereas, for larger number of layers the value of Δ_{opt} is much less than 1. Hence, N_Δ is very large. Therefore, in order to compute Δ_{opt} large number of harmonics have to be considered. In [7], Δ_{opt} is represented in terms of a ratio of two series summations but the harmonic number till which the summations are to be carried out is not mentioned.

Differentiation of (15) with respect to Δ, gives an implicit equation involving Δ_{opt}, (16)

$$\frac{-1}{\Delta_{opt}^2}\left[k_1\left(\frac{2.5}{\Delta_{opt}}\right)^{2k_2}(2k_2+1) + k_3\right] \\ + \left(\frac{5p^2-1}{45}\right)\left(3.125 + 3k_4\Delta_{opt}^2\right) \\ + \left(\frac{2p^2+1}{3}\right)\left[k_6\left(\frac{2.5}{\Delta_{opt}}\right)^{2k_7}\left(\frac{2k_7}{\Delta_{opt}}\right)\right] = 0 \quad (16)$$

Δ_{opt} can be obtained by solving, (16) numerically. Δ_{opt} can also be computed using MATLAB's optimization toolbox.

$$\Delta_{opt} = m_1 p^{m_2} + m_3 \quad (17)$$

Table II, gives coefficients of a curve fitting formula shown in (17), to compute Δ_{opt} for specific duty ratios, for 1-25 layers.

TABLE II
COEFFICIENTS FOR COMPUTATION OF Δ_{opt}

Coefficients	$D = 0.25$	$D = 0.5$	$D = 0.6$	$D = 0.75$	$D = 0.8$	$D = 1$
m_1	1.134	1.394	1.402	1.518	1.529	1.553
m_2	-1.112	-1.061	-0.993	-1.026	-1.022	-1.016
m_3	0.01386	0.0107	-0.001025	0.005249	0.004716	0.003636

IV. Validation of Results

A duty cycle modulated square waveform of duty ratio $D = 1$ is considered. A MATLAB code is written to compute P_{pu} using (7) by considering 20000 harmonics for 1-25 layers at Δ=0.1. P_{pu} is also computed by solving (15) and Fig. 4 shows a comparison of both methods. Then Δ_{opt} is obtained by searching the minimum value of P_{pu}. Δ_{opt} is also found by

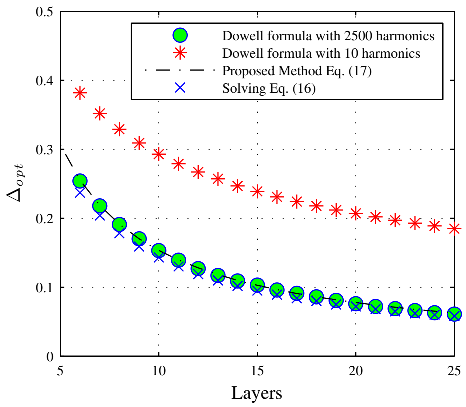

Fig. 5. Comparison of different methods for computation of Δ_{opt}.

solving (16) and by solving (17) and a comparison of all three methods is shown in Fig. 5. If (7) is used to compute Δ_{opt} graphically for $p = 2$, Δ_{opt} is 0.764, so only 11 harmonics are required to compute Δ_{opt}. But for $p = 8$, Δ_{opt} is 0.191, so 172 harmonics are required to compute Δ_{opt}. If only 10 harmonics are considered then for $p = 8$, Δ_{opt} is 0.329 leading to an error of 72%. On the other hand, if (16) is solved using MATLAB for $p = 8$, Δ_{opt} is 0.175, leading to an error of 8.4%. Finally if (17) is used then Δ_{opt} is 0.1893, leading to an error of 0.91%. This confirms the advantage and efficiency of the method proposed in this paper. Once Δ_{opt} is determined for a given layer p, optimal value of d can be found. Using (15) it is possible to obtain an estimate of power loss for a given layer p at the optimal thickness.

V. Conclusion

The copper loss in a multi-layer transformer winding for a particular duty-cycle modulated current waveform at a given frequency depends on the number of layers and the thickness of each layer. For a given number of layers, there is an optimal thickness that results in minimum copper loss.

It has been shown in this paper that Fourier series method requires consideration of a large number of harmonics leading to computational difficulty of power loss and determination of optimal thickness. In contrast, an approximate closed form expression for power loss has been derived in this paper that does not involve any series summation. This paper presents a simple expression to compute the optimal thickness for a given number of layers which can be used to compute the power loss. Through numerical computation this approach has been validated. A designer can simply use these formulas instead of writing complex computationally demanding MATLAB codes to obtain optimal thickness and estimate the copper loss.

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References

[1] P. Dowell, "Effects of eddy currents in transformer windings," *Electrical Engineers, Proceedings of the Institution of*, vol. 113, no. 8, pp. 1387 –1394, august 1966.

[2] P. Venkatraman, "Winding eddy current losses in switch mode power transformers due to rectangular wave currents," in *Proceedings of Powercon*, vol. 11, 1984, pp. 1–11.

[3] S. Crepaz, "Eddy-current losses in rectifier transformers," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-89, no. 7, pp. 1651 –1656, sept. 1970.

[4] W. Hurley, E. Gath, and J. Breslin, "Optimizing the ac resistance of multilayer transformer windings with arbitrary current waveforms," *Power Electronics, IEEE Transactions on*, vol. 15, no. 2, pp. 369 –376, mar 2000.

[5] M. Dale and C. Sullivan, "General comparison of power loss in single-layer and multi-layer windings," in *Power Electronics Specialists Conference, 2005. PESC '05. IEEE 36th*, june 2005, pp. 582 –589.

[6] X. Nan and C. Sullivan, "An improved calculation of proximity-effect loss in high-frequency windings of round conductors," in *Power Electronics Specialist Conference, 2003. PESC '03. 2003 IEEE 34th Annual*, vol. 2, june 2003, pp. 853 – 860 vol.2.

[7] J. Breslin and W. Hurley, "Derivation of optimum winding thickness for duty cycle modulated current waveshapes," in *Power Electronics Specialists Conference, 1997. PESC '97 Record., 28th Annual IEEE*, vol. 1, jun 1997, pp. 655 –661 vol.1.

Appendix

The dc resistance of a single layer of a transformer containing multiple turns at a thickness equal to the skin depth of the conductor as shown in Fig. 6 is

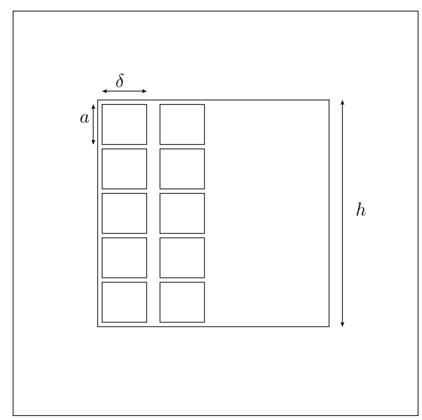

Fig. 6. Multi-turn Multi-layer foil winding of a transformer.

$$R_{dc}|_{d=\delta} = \frac{N_l \rho (MLT)}{a\delta} \tag{A.1}$$

For a transformer, containing p layers, the $R_{dc}|_{d=\delta}$ takes the form,

$$R_{dc}|_{d=\delta} = \frac{N_l \rho (MLT) p}{a\delta} \tag{A.2}$$

Substituting a as $\eta h/N_l$ and $N_l p$ as N,

$$R_{dc}|_{d=\delta} = \frac{N^2 \rho (MLT)}{h\delta p\eta} \tag{A.3}$$