Sensitivity in Translation Averaging

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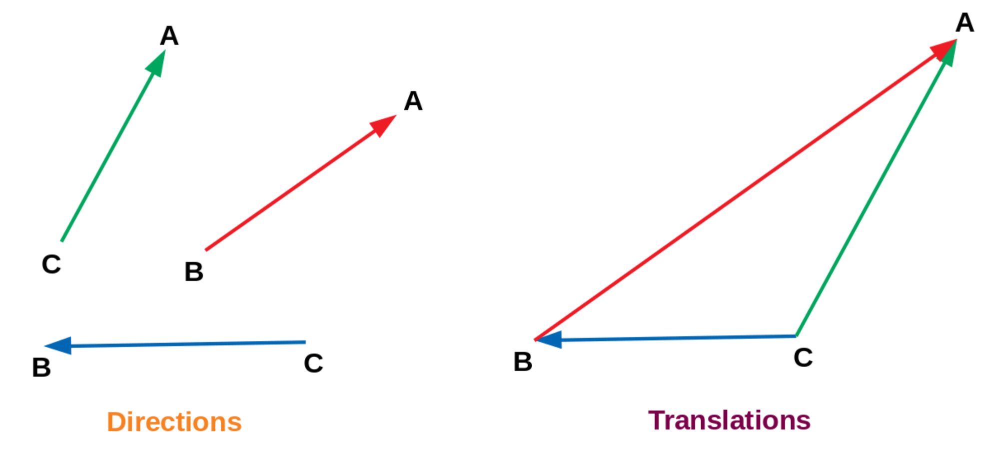
NEURAL INFORMATION PROCESSING SYSTEMS

Translation Averaging in 3D Computer Vision



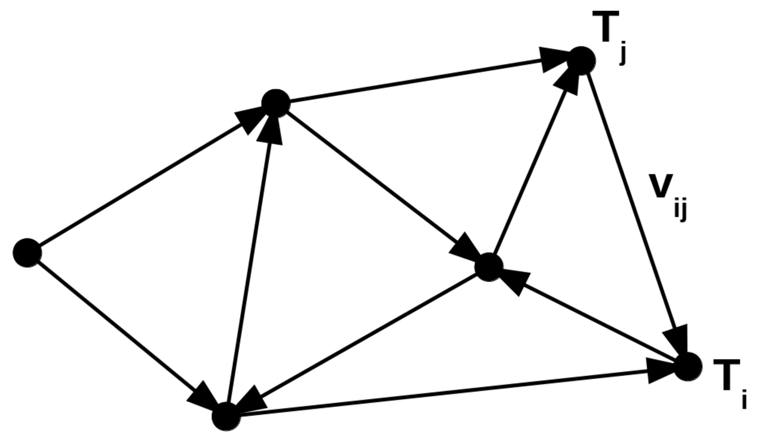
Solving for **translations** given **directions**.

- Input: Directions.
- Output: Translations.
- Solution upto scale and origin.
- Assumes known 3D rotations.
- Application: Structure-from-Motion.

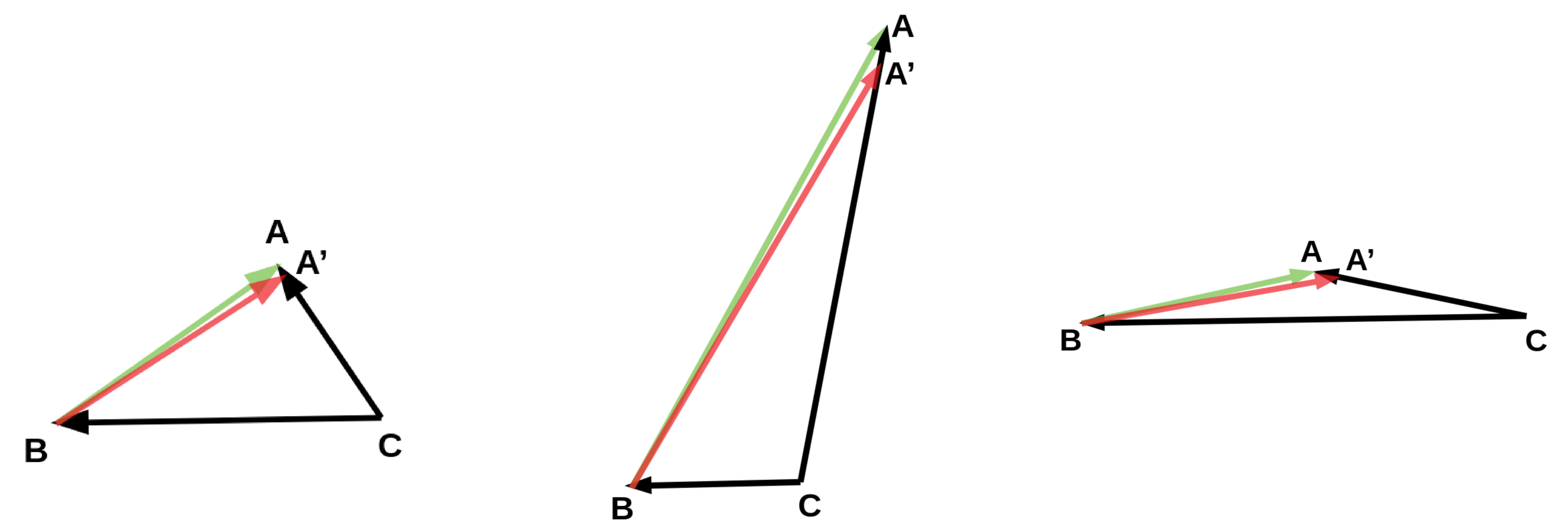
Problem Setup

- Represented as a network: $\mathcal{G} = (\mathcal{V}, \mathcal{E})$.
- Nodes: absolute translations $\mathbf{T}_i \in \mathbb{R}^3, i \in \mathcal{V}$.
- Edges: directions $\mathbf{v}_{ij} \in \mathbb{S}^2, (i,j) \in \mathcal{E}$.
- Called bearing-based network.
- Consistency equation with scale (s) and origin ($\mathbf{T}_O$) ambiguity:

$$\mathbf{v}_{ij} = \frac{\mathbf{T}_j - \mathbf{T}_i}{\|\mathbf{T}_j - \mathbf{T}_i\|} = \frac{s(\mathbf{T}_j - \mathbf{T}_O) - s(\mathbf{T}_i - \mathbf{T}_O)}{\|s(\mathbf{T}_j - \mathbf{T}_O) - s(\mathbf{T}_i - \mathbf{T}_O)\|}.$$



Main Considerations



Well-conditioned triangle | Ill-conditioned triangle (Type-I) | Ill-conditioned triangle (Type-II)

Conditioning with respect to perturbations in directions (green $\rightarrow$ red $\implies$ A $\rightarrow$ A').

- Parallel rigidity [1]: Deals with the uniqueness of the solution.
- Outlier detection/suppression [2, 5]: Deals with gross errors in input directions.
- **Sensitivity [Ours]: Deals with small perturbations in input directions.**

Theoretical Results: Smallest Solvable Network

Formulation: Consider the smallest solvable bearing-based network i.e. a network $\mathcal{G}_\Delta$ of 3 nodes, $\mathcal{V}_\Delta = \{1,2,3\}$, with all possible edges, $\mathcal{E}_\Delta = \{(1,2),(2,3),(3,1)\}$. Least squares problem to estimate scales: $\min_{\mathbf{s}} \|\mathbf{V}\mathbf{s}\|^2$ s.t. $\|\mathbf{s}\|^2 = 1$.
Solution: Eigen vector corresponding to smallest eigenvalue of $\mathbf{V}^\mathbf{T}\mathbf{V}$.

Theorem 1: For a set of consistent directions, $\mathbf{v}_{ij}, (i,j) \in \mathcal{E}_\Delta$, in $\mathcal{G}_\Delta$, the absolute change in any eigenvalue of $\mathbf{V}^\mathbf{T}\mathbf{V}$, denoted as $|\delta\lambda|$, when the directions $\mathbf{v}_{ij}$ are perturbed by small rotations $\delta\mathbf{R}_{ij}$, with $\mathbf{n}_{ij}$ and $\delta\theta_{ij} > 0$ being the rotation axis and angle, is bounded by

$$|\delta\lambda| \leq \sum_{(k,i,j) \in TI(\Delta)} \delta\theta_{ij} \cdot \|\mathbf{v}_{ki}^{\mathbf{n}_{ij}\perp}\| \cdot \|\mathbf{v}_{kj}^{\mathbf{n}_{ij}\perp}\| \cdot \frac{\left|\sin\phi_{(k,i),(k,j)}^{\mathbf{n}_{ij}\perp}\right|}{\sin^2\phi_{(k,i),(k,j)}} \cdot$$
$$\left[\left(1 + (\mathbf{v}_{ik}^T\mathbf{v}_{jk})^2\right)\left((\mathbf{v}_{ij}^T\mathbf{v}_{ik})^2 + (\mathbf{v}_{ij}^T\mathbf{v}_{jk})^2\right) - 4 \cdot \mathbf{v}_{ik}^T\mathbf{v}_{jk} \cdot \mathbf{v}_{ij}^T\mathbf{v}_{ik} \cdot \mathbf{v}_{ij}^T\mathbf{v}_{jk}\right]^{\frac{1}{2}},$$

where $TI(\Delta) = \{(1,2,3),(2,3,1),(3,1,2)\}$, $\mathbf{v}^{\mathbf{n}_{ij}\perp}$ is the component of $\mathbf{v}$ orthogonal to $\mathbf{n}_{ij}$, $\phi_{(k,i),(k,j)}^{\mathbf{n}_{ij}\perp}$ is the angle between $\mathbf{v}_{ki}^{\mathbf{n}_{ij}\perp}$ and $\mathbf{v}_{kj}^{\mathbf{n}_{ij}\perp}$, and $\phi_{(k,i),(k,j)}$ is the angle between $\mathbf{v}_{ki}$ and $\mathbf{v}_{kj}$.

Angle Matrix for $\mathcal{G}_\Delta$: $\mathbf{A}_{\mathcal{G}_\Delta} \in \mathbb{R}^{3\times3}$ ($\phi_{(k,i),(k,j)}$: angle between $\mathbf{v}_{ki}$ and $\mathbf{v}_{kj}$)

$$\mathbf{A}_{\mathcal{G}_\Delta} = \begin{bmatrix} \phi_{(1,2),(1,2)} & \phi_{(2,1),(2,3)} & \phi_{(1,2),(1,3)} \\ \phi_{(2,3),(2,1)} & \phi_{(2,3),(2,3)} & \phi_{(3,2),(3,1)} \\ \phi_{(1,3),(1,2)} & \phi_{(3,1),(3,2)} & \phi_{(3,1),(3,1)} \end{bmatrix} = \begin{bmatrix} 0 & \phi_{(2,1),(2,3)} & \phi_{(1,2),(1,3)} \\ \phi_{(2,1),(2,3)} & 0 & \phi_{(3,2),(3,1)} \\ \phi_{(1,2),(1,3)} & \phi_{(3,2),(3,1)} & 0 \end{bmatrix}.$$

Theorem 2: Consider the bearing based-network of 3 nodes and 3 edges, $\mathcal{G}_\Delta = (\mathcal{V}_\Delta, \mathcal{E}_\Delta)$, and the corresponding angle matrix $\mathbf{A}_{\mathcal{G}_\Delta}$. The conditioning of the matrix $\mathbf{A}_{\mathcal{G}_\Delta}$ signifies the skewness of the triangle formed using the directions in $\mathcal{E}_\Delta$.

Theoretical Results: General Network Consisting of Triplets

Extending angle matrix:
- **Angle Matrix for $\mathcal{G}$: $\mathbf{A}_\mathcal{G}$** $\in \mathbb{R}^{M\times M}$ (M is the number of edges in $\mathcal{G}$).
- **Assumption:** All edges in $\mathcal{G}$ are a part of at least one triplet.

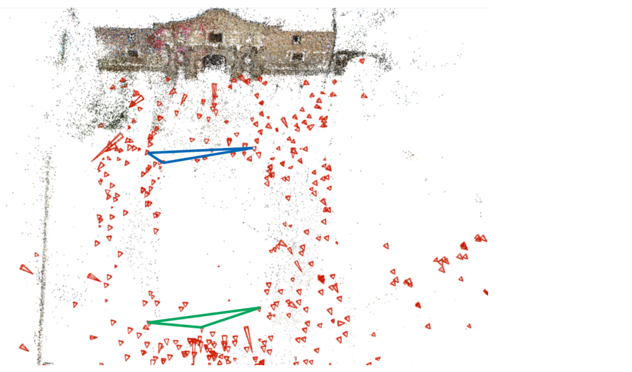
Conditioning of Translation Averaging: Defined as condition number of the angle matrix $\mathbf{A}_\mathcal{G}$.

Theorem 3: Consider a bearing-based network $\mathcal{G}$, with all edges contributing to triplets. The angle matrix $\mathbf{A}_\mathcal{G}$, corresponding to $\mathcal{G}$, is well conditioned if the minimum angle (or equivalently all the angles) in all the triangles formed by the triplets are sufficiently large.

Dealing with uniqueness of solution:
- Theorem 3 does not assume parallel rigidity of the network $\mathcal{G}$.
- Construct triplet network: $\mathcal{G}_T = (\mathcal{V}_T, \mathcal{E}_T)$, Nodes $\mathcal{V}_T$ denote a triplet in $\mathcal{G}$, Edges $\mathcal{E}_T$ connect the nodes if an edge is common between the triplets in $\mathcal{G}$.

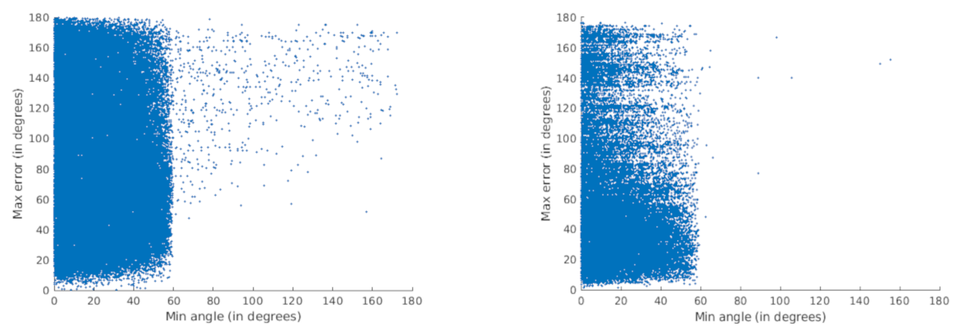
Theorem 4: Given a bearing-based network $\mathcal{G}$, with all edges contributing to triplets forming triangles, and its corresponding triplet network $\mathcal{G}_T$, the maximal parallel rigid component of $\mathcal{G}$ can be determined by the edges in $\mathcal{G}$ contributing to the largest connected component of $\mathcal{G}_T$.

Analysis of Real Data



Alamo reconstruction [4] using COLMAP [3].

Blue triangle: Ill-conditioned (Type-I).
Green triangle: Ill-conditioned (Type-II).



Piccadilly | Tower of London

Maximum error of directions vs minimum angle between edges.

Abundance of skewed triangles.

Outlier presence $\neq$ Skewed triangle.

Experimental Results

Dataset	$\#N_{rem}$	$\#M_{rem}$	Mean-ATE ↓		RMS-ATE ↓		Removed Node Errors		P_{tri} ↑		BA_{iters} ↓	
			w/o filter	w/ filter	w/o filter	w/ filter	Mean	RMS	w/o filter	w/ filter	w/o filter	w/ filter
Alamo	12	197	4.7	**4.5**	11.1	**10.5**	22.9	39.5	142	**144**	73	**54**
Ellis Island	8	69	23.2	**22.1**	**50.7**	51.8	98.6	118.1	57	**58**	40	**33**
Gendarmenmarkt	24	257	50.6	**40.9**	77.8	**61.1**	149.7	190.8	101	**104**	31	**21**
Madrid Metropolis	24	197	13.9	**12.7**	29.4	**26.1**	54.2	61.2	46	**49**	**66**	83
Montreal Notre Dame	4	147	4.4	**4.3**	10.0	**9.9**	31.2	34.1	115	**118**	59	**37**
NYC Library	13	211	6.5	**5.2**	15.8	**12.6**	28.7	38.1	77	**79**	100	**74**
Notre Dame	3	108	**3.3**	**3.3**	**6.4**	**6.4**	6.9	7.0	**376**	375	34	**22**
Piazza del Popolo	18	268	**8.0**	**8.0**	**13.0**	13.1	15.0	17.3	91	**94**	**34**	44
Piccadilly	61	901	5.3	**5.1**	11.0	**10.8**	20.8	29.2	231	**234**	53	**47**
Roman Forum	41	394	12.8	**10.4**	27.0	**19.6**	65.4	100.5	**196**	**196**	31	**21**
Tower of London	17	141	15.6	**14.5**	32.2	**30.3**	63.7	90.3	107	**110**	89	**85**
Union Square	26	232	14.5	**10.6**	24.8	**18.1**	34.5	49.9	57	**56**	122	**52**
Vienna Cathedral	29	282	10.2	**10.2**	**17.8**	18.4	21.4	27.5	219	**222**	52	**25**
Yorkminster	34	363	20.4	**19.3**	29.5	**28.3**	44.0	51.4	**175**	168	58	**33**

Absolute translations errors (ATE) (in m), points triangulated ($P_{tri} \times 10^3$) and bundle adjustment iterations (BA_{iters}) on 1DSfM [4] datasets using BATA [5]. Removed Node Errors: Errors of removed nodes in the unfiltered network.

Conclusion

This work deals with sensitivity in translation averaging under input uncertainty;
- studies sensitivity in estimating edge scales suggesting skewed triangles are unstable,
- defines conditioning of translation averaging problem and provides a sufficient criterion to ensure it is well-conditioned,
- proposes an efficient algorithm to remove skewed triangles from the network while ensuring parallel rigidity,
- demonstrates the effectiveness of the proposed filter with better absolute translations, more 3D points triangulated and faster convergence of bundle adjustment for filtered networks.

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