Adaptive Annealing for Robust Averaging

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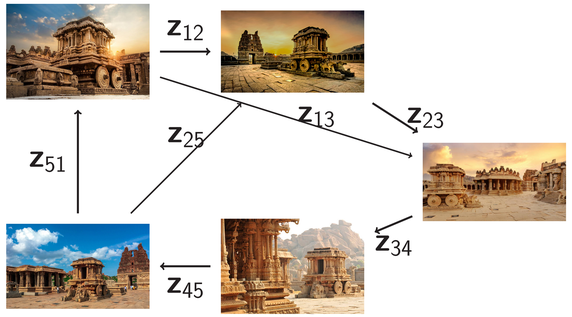
Introduction



- Global pipelines for large-scale 3D reconstruction entail solving the global poses of cameras given the relative poses between pairs of cameras.
- **Averaging**: Given a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and a set of outlier-prone pairwise observations $\{z_{ij} \mid (i,j) \in \mathcal{E}\}$, where z_{ij} is a function of x_i and x_j, estimate the node values $\{x_i \mid i \in \mathcal{V}\}$.
- Minimization of a robust cost to mitigate outliers.

$$f_\sigma(x) = \sum_{ij \in E} \rho_\sigma(\|r_{ij}(x_i, x_j; z_{ij})\|) \tag{1}$$

Graduated Non-Convexity (GNC)

- Solve Eq. 1 by minimizing a sequence of increasingly hard-to-solve robust costs $\rho_\sigma(\cdot)$, $\sigma = \sigma_0 > \sigma_1 > \sigma_2 > \cdots > \sigma_{min}$.
- Adaptive $\{\sigma_k\}$ schedule [4] $\Rightarrow$ Good accuracy-speed tradeoff.
- **Core Idea**: Track the positive-definiteness of the Hessian (H) to determine the next σ.
- We exploit the **graph Laplacian structure** of the Hessian matrices in averaging problems.

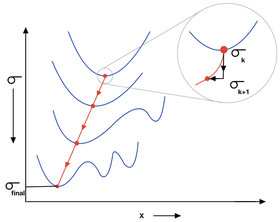


Figure: Annealing σ

$$H(\sigma) = \begin{bmatrix} D_1(\sigma) & -W_{12}(\sigma) & -W_{13}(\sigma) & \cdots & -W_{1N}(\sigma) \\ -W_{21}(\sigma) & D_2(\sigma) & -W_{23}(\sigma) & \cdots & -W_{2N}(\sigma) \\ -W_{31}(\sigma) & -W_{32}(\sigma) & D_3(\sigma) & \cdots & -W_{3N}(\sigma) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -W_{N1}(\sigma) & -W_{N2}(\sigma) & -W_{N3}(\sigma) & \cdots & D_N(\sigma) \end{bmatrix}$$

where $D_i(\sigma) = \sum_{j=1, j\neq i}^{N} W_{ij}(\sigma)$, and $W_{ij}(\sigma)$ is the weight matrix corresponding to z_{ij}.

Proposed Annealing Scheme

Laplacian Property-1: *The Laplacian matrix $H(\sigma)$ is positive semidefinite if $W_{ij}(\sigma) \succcurlyeq \mathbf{0}$, $\forall (i,j) \in \mathcal{E}$.*

$\Downarrow$

Anneal σ to ensure $W_{ij}(\sigma) \succcurlyeq \mathbf{0}\ \forall (i,j) \in \mathcal{E}$. Once this is not possible, reduce σ so that at most p% of edges have $W_{ij} \not\succcurlyeq \mathbf{0}$.

Percentile Approach:

- Let $\sigma_{ij} = \text{argmin}_\sigma\{\lambda_{min}(W_{ij}(\sigma)) = 0\}$. Set σ as the $(100-p)^{th}$ percentile of $\{\sigma_{ij}\}$.
- Increase p in fixed steps until $p = 50$ or $\sigma \geq \sigma_{min}$.

Advantages

- σ_{ij} is a closed-form function of r_{ij}, enabling efficient σ computation for the next GNC stage.
- σ_{min} can be adaptively chosen as the k^{th} percentile of $\{\sigma_{ij}\}$, effective for up to k% outliers.
- Efficiently detect $\lambda_{min}(H) > 0$ via power iteration by assessing the eigenvector's proximity to the all-ones vector.

$\Uparrow$

Laplacian Property-2: *A Laplacian with no negative eigenvalue has its smallest eigenvector as the all-ones vector.*

Illustrative Problems

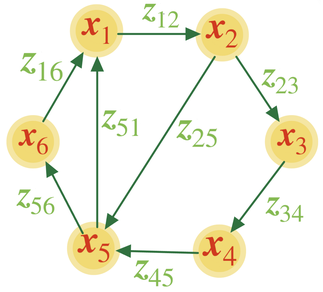


Figure: Vector Averaging

Input: Relative **Displacements** (z_{ij})

$$\min_{\{x_i\}} \sum_{ij \in E} \rho_\sigma(\|x_i - x_j - z_{ij}\|)$$

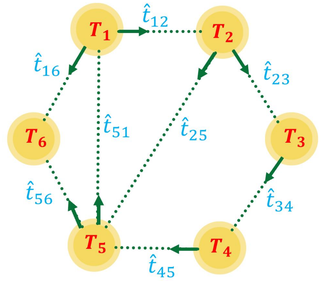


Figure: Translation Averaging

Input: Relative **Directions** ($\hat{t}_{ij}$)

$$\min_{\{T_i\},\{d_{ij}\}} \sum_{ij \in E} \rho_\sigma(\|T_i - T_j - d_{ij}\hat{t}_{ij}\|)$$

Output: **Camera/Sensor Locations** (x_i, T_i)

Results

Superior performance on synthetic datasets for varying outlier fractions and graph densities.

	IRLS			GNC ($\delta \downarrow$)			GNC ($\delta \uparrow$)			SE-Sync [3]			GNCp		
lr1	2.15	2.04	0.05	**1.93**	1.60	0.65	2.00	1.66	0.15	2.15	2.11	7.7	**1.93**	**1.59**	0.22
lr2	1.39	1.32	0.27	**1.25**	**1.31**	3.64	1.31	**1.31**	0.53	1.55	1.53	28	**1.25**	**1.31**	3.11
of1	1.87	1.89	0.06	1.81	1.80	0.52	1.87	1.88	0.10	1.84	1.88	9.5	**1.80**	**1.79**	0.18
of2	2.15	2.20	0.18	**2.12**	1.94	1.68	2.17	2.24	0.34	2.16	2.24	14	**2.12**	**1.93**	0.95

Table: Vector averaging on Augmented ICL-NUIM dataset (*lr*:livingroom, *of*:office): {Mean | median errors | time taken} of the baselines

Dataset	N	M	Mean | Median errors ↓									
			LUD [2]		1DSfM [5]		ShapeFit [1]		BATA [6]		GNCp-LUD	
Alamo	586	81437	2.7	0.5	4.E+03	0.5	20.9	0.6	2	0.6	**1.9**	0.5
Ellis Island	229	14728	6.9	3.6	2.E+04	3.4	35	6.1	6.7	3.2	**6.6**	3.1
Gendarmenkmart	686	27145	31.2	11.3	1.E+04	11.4	36.8	14.8	31.3	11.4	**27.8**	11.3
Madrid Metropolis	325	11995	8.4	1.9	2.E+05	2	24	2.8	6.9	**1.6**	**6.4**	1.6
Montreal Notre Dame	461	45737	0.9	0.5	6.E+03	0.5	2.2	0.9	**0.8**	0.5	**0.8**	0.5
NYC Library	337	14365	2.2	0.8	4.E+04	0.8	2.9	1.2	**2.1**	1.7	2.3	**0.8**
Notre Dame	552	80647	1.2	0.3	1.E+05	0.3	6.8	0.5	**1**	0.2	1.1	**0.2**
Piazza del Popolo	334	20974	3.8	2.8	8.E+03	1.7	20.1	**1.6**	**3.4**	2	3.5	2.3
Piccadilly	2362	201600	**2.8**	1.3	2.E+04	1.2	19.8	1.3	3.2	**1.1**	3.2	1.2
Roman Forum	1069	54207	11.9	3.3	1.E+05	1.8	18.5	6.3	8.3	2	**7.3**	1.7
Tower of London	474	19252	14.9	3.2	1.E+05	3	15.3	6.6	12.1	2.9	**10.8**	3
Union Square SF	1369	123081	2.4	0.8	5.E+04	1.1	6.7	1.6	2.6	0.8	**2.3**	**0.7**
Vienna Cathedral	876	52274	6.2	0.5	4.E+05	0.5	10.4	1.1	6.3	0.5	**5.8**	0.5
Yorkminster	1017	45117	10.2	4.7	2.E+04	4.3	14.8	6.3	8.3	3.2	**8**	4.2

Table: Translation Averaging on real data: Mean | Median errors (in metres) of the baselines

Conclusion

- In averaging problems, we exploit the graph Laplacian structure of the Hessian matrices to devise a principled method to adaptively anneal the scale in Graduated Non-Convexity (GNC).
- We demonstrate the superior accuracy and efficiency of our approach compared to the state-of-the-art on two problems: Vector averaging, Translation averaging.

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References

[1] Goldstein, T., Hand, P., Lee, C., Voroninski, V., Soatto, S.: Shapefit and shapekick for robust, scalable structure from motion. In: European Conference on Computer Vision. pp. 289–304. Springer (2016)

[2] Ozyesil, O., Singer, A.: Robust camera location estimation by convex programming. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. pp. 2674–2683 (2015)

[3] Rosen, D.M., Carlone, L., Bandeira, A.S., Leonard, J.J.: Se-sync: A certifiably correct algorithm for synchronization over the special euclidean group. The International Journal of Robotics Research **38**(2-3), 95–125 (2019)

[4] Sidhartha, C., Manam, L., Govindu, V.M.: Adaptive annealing for robust geometric estimation. In: Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR). pp. 21929–21939 (June 2023)

[5] Wilson, K., Snavely, N.: Robust global translations with 1dsfm. In: European Conference on Computer Vision. pp. 61–75. Springer (2014)

[6] Zhuang, B., Cheong, L.F., Lee, G.H.: Baseline desensitizing in translation averaging. In: Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition. pp. 4539–4547 (2018)

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