# Global Mesh Denoising with Fairness

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## Introduction

### 3D Noise

- Since raw 3D data contains **significant amounts of noise**, all 3D reconstruction or application pipelines need to carry out an important **denoising step**.
- We assume the **noise to be present in all directions** about a 3D point.

### Classes of mesh denoising methods:

- **Local methods** - The correction for the noisy mesh is applied locally, resulting in approaches that are applied iteratively, *e.g.* Field, 1988; Taubin, 2001; Fleishman *et al.*, 2003; Sun *et al.*, 2007; Zheng *et al.*, 2011.
- **Global methods** - A global cost function is optimised resulting in solving sparse equations *e.g.* Desbrun *et al.*, 1999; Ohtake *et al.*, 2002; Ji *et al.*, 2005; Nealen *et al.*, 2006; Liu *et al.*, 2007; He and Schaefer, 2013; Cheng *et al.*, 2014; Wang *et al.*, 2014; Zhang *et al.*, 2015.

### Limitations of existing methods:

- **Assume** the noise to be restricted to the direction of the surface normal.
- Correct for the position of a mesh vertex by **moving** it along this normal direction.
- The noise component in the tangent plane leads to a **severe distortion of the face shapes** including **face flipping**.
- Local methods have difficulty in defining the **number of times the filter** to be applied for optimal denoising.
- Significant **implicit volume shrinkage**.
- **Smoothing over surface features** such as edges and corners.

## Our Contribution

**A novel global 3D mesh denoising method** with the following **properties**:

- **Global** formulation.
  - Minimises **sparse, quadratic** cost functions.
  - Yields efficient solutions.
- We allow for a vertex correction **in all directions** while enforcing a **novel face fairness penalty** that preserves face shapes in the denoised mesh.
- It has good **volume** and **sharp feature preserving** properties.

## Proposed Method

### Two steps:

- **Normal Mollification** - A **global formulation** depending only on the **variance of the noise** for mollification.
- **Vertex Correction** - A **robust global anisotropic formulation** for vertex correction with face fairness.

### Normal Mollification:

- **Global** in formulation and depends only on the **variance of the noise**.
- For each face normal $\mathbf{n}_i^F$, our cost function contains two terms:
  - ✓ A **quadratic penalty** data term $d_o\left(\hat{\mathbf{n}}_i^F, \mathbf{n}_i^F\right)$, and
  - ✓ A **weighted quadratic anisotropic smoothness term** $\sum_{j\in\mathcal{N}_F(i)} w_{ij}^2 d_s\left(\hat{\mathbf{n}}_j^F, \hat{\mathbf{n}}_i^F\right)$.
- Minimise

$$\sum_{i=1}^{N_F} d_o\left(\hat{\mathbf{n}}_i^F, \mathbf{n}_i^F\right) + \lambda_N \sum_{i=1}^{N_F} \sum_{j\in\mathcal{N}_F(i)} w_{ij}^2 d_s\left(\hat{\mathbf{n}}_j^F, \hat{\mathbf{n}}_i^F\right) \text{ subject to } ||\hat{\mathbf{n}}_i^F||^2 = 1,\ i = 1, 2, \cdots, N_F$$

where $\lambda_N$ is a regularising parameter depending on the noise variance and the face neighbourhood operator $\mathcal{N}_F(i)$ is the set of faces sharing a common vertex with $\mathbf{f}_i$ and

$$w_{ij}\left(\hat{\mathbf{n}}_j^F, \hat{\mathbf{n}}_i^F\right) = \begin{cases} \left(\hat{\mathbf{n}}_j^{F,T}\hat{\mathbf{n}}_i^F - t\right) & \text{if } \hat{\mathbf{n}}_j^{F,T}\hat{\mathbf{n}}_i^F > t \\ 0 & \text{otherwise} \end{cases}$$

where $t$ is a threshold.

## Proposed Method (Contd.)

### Vertex Correction:

- Our vertex correction step is **global** in nature and depends only on the **variance of the noise**.
- We construct a global cost function $C_V(\hat{\mathbf{V}})$ which has three terms:
  - ✓ A **quadratic** data term $d_o^V\left(\hat{\mathbf{V}}, \mathbf{V}\right) = \left\|\hat{\mathbf{V}} - \mathbf{V}\right\|_2^2$,
  - ✓ A **weighted quadratic anisotropic Laplacian term** $d_s^V(\hat{\mathbf{V}})$, and
  - ✓ A novel **face fairness term** $d_f^V(\hat{\mathbf{V}}, \mathbf{V})$.
- Minimise

$$C_V(\hat{\mathbf{V}}) = d_o^V(\hat{\mathbf{V}}, \mathbf{V}) + \lambda_V d_s^V(\hat{\mathbf{V}}) + \eta d_f^V(\hat{\mathbf{V}}, \mathbf{V})$$

where $\lambda_V$ and $\eta$ are parameters depending only on the type and amount of noise.

- **Face fairness** term **ensures that triangular faces** do not become **skinny or folded**.

### Our global Laplacian

- For each vertex $\mathbf{v}_i$ the Laplacian operator is

$$\mathbf{L}_i(\mathbf{v}_i) = \sum_{j\in\mathcal{N}_V(i)} \frac{a_j b_j}{(1+b_j)\sum_{j\in\mathcal{N}_V(i)} a_j} \left(\mathbf{n}_j^F \mathbf{n}_j^{F,T}\right) \cdot \left(\mathbf{v}_i - \frac{(\mathbf{v}_{j_1} + \mathbf{v}_{j_2} + \mathbf{v}_{j_3})}{3}\right)$$

where $\mathbf{n}_j^F$ is the normal of face $\mathbf{f}_j$ corresponding to $\mathbf{v}_i$ *i.e.* $\mathcal{N}_V(i)$ is the set of 1-ring neighbouring faces to $\mathbf{v}_i$ and $\mathbf{v}_{j_1}$, $\mathbf{v}_{j_2}$, $\mathbf{v}_{j_3}$ are vertices of $\mathbf{f}_j$. $\Delta\mathbf{v}_{ij} = \frac{\mathbf{v}_{j_1}+\mathbf{v}_{j_2}+\mathbf{v}_{j_3}}{3} - \mathbf{v}_i$.

- Anisotropic bilateral weights:

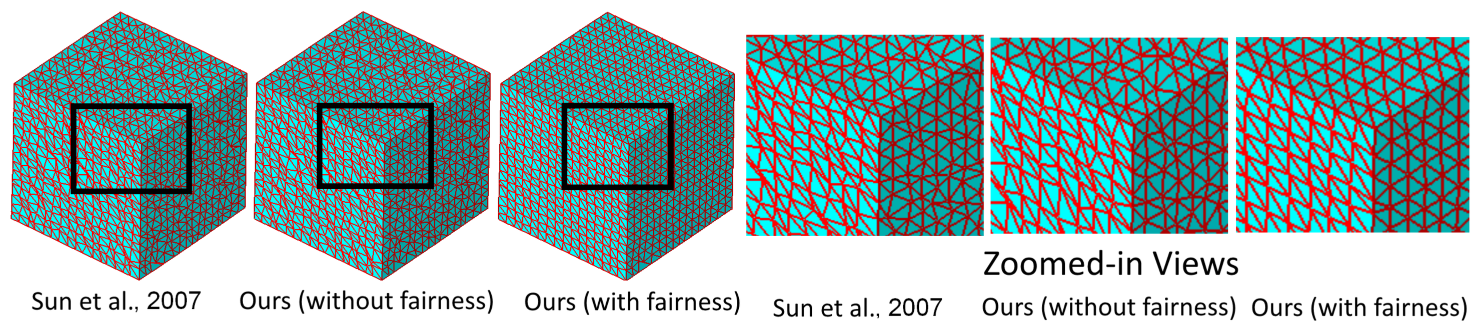
$$a_j = \exp\left(-\frac{\left(\mathbf{n}_j^T \Delta\mathbf{v}_{ij}\right)^2}{2\sigma_1^2}\right), b_j = \exp\left(-\frac{\|\Delta\mathbf{v}_{ij}\|_2^2}{2\sigma_2^2}\right).$$

### Face fairness penalty

- Denoising in the tangent plane about a vertex.
- The face fairness penalty for a single denoised vertex $\hat{\mathbf{v}}_i$, is

$$d_f^V(\hat{\mathbf{v}}_i) = \left\|r_i(\mathbf{I} - \mathbf{n}_i^V \mathbf{n}_i^{V,T})(\hat{\mathbf{v}}_i - \mathbf{v}_{c,i})\right\|_2^2,\ r_i = \begin{cases} 0 & \text{if } \mathbf{v}_i \in \mathbf{V}^B \text{ or } \beta < 0 \\ \beta & \text{otherwise.} \end{cases}$$

- $\beta = \min_{p,q\in\mathcal{N}_V(i)} (\mathbf{n}_p^{F,T}\mathbf{n}_q^F - \delta)$ and $\mathbf{v}_{c,i}$ - centroid of the 1-ring face neighbourhood around the vertex $\mathbf{v}_i$.



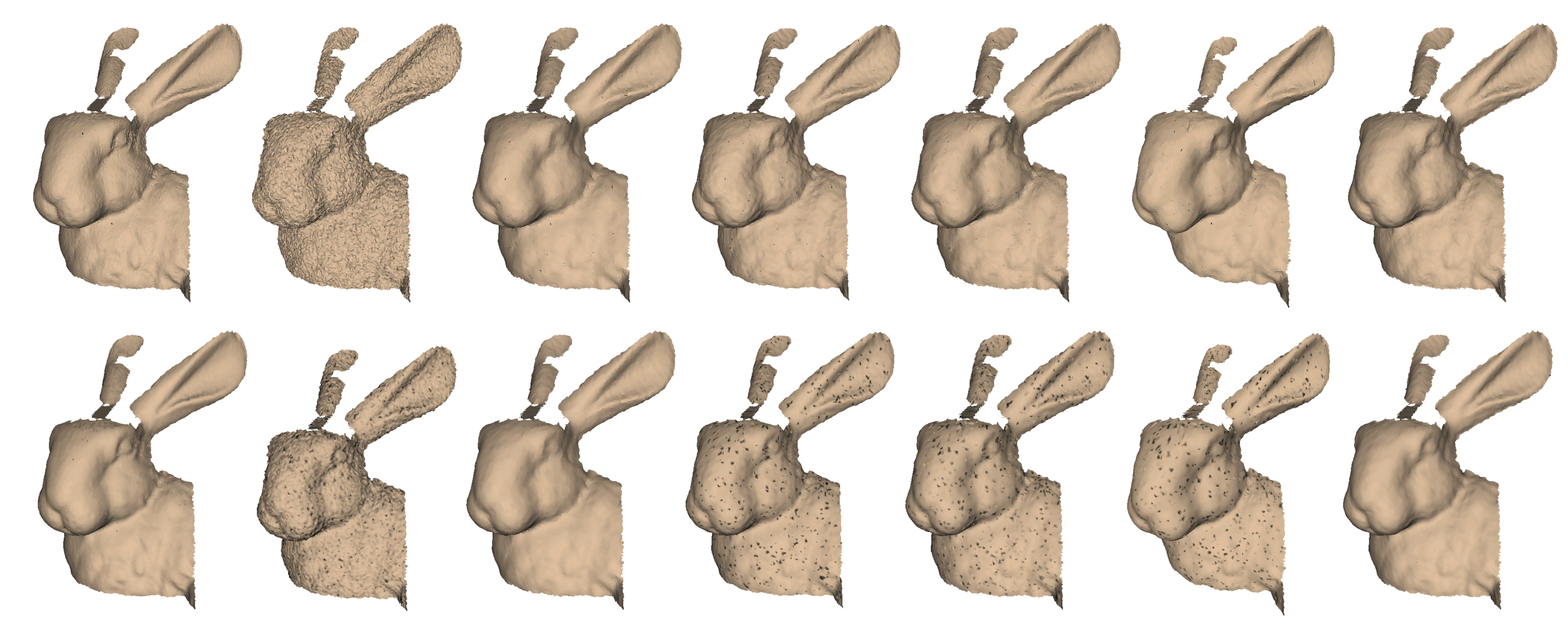

Face quality of denoised mesh of a cube corrupted with isotropic Gaussian noise ($\sigma = 0.15 \times$ mean edge length).

### Overall vertex correction equation:

$$\hat{\mathbf{V}} = \left(\mathbf{I} + \lambda_V \mathbf{L}^T\mathbf{L} + \eta\mathbf{K}^T\mathbf{K}\right)^{-1}\left(\mathbf{V} + \eta\mathbf{K}^T\mathbf{K}\mathbf{V}_c\right)$$

where $\mathbf{K}$ and $\mathbf{V}_c$ are formed from the face fairness term.

## Result: Synthetic Data



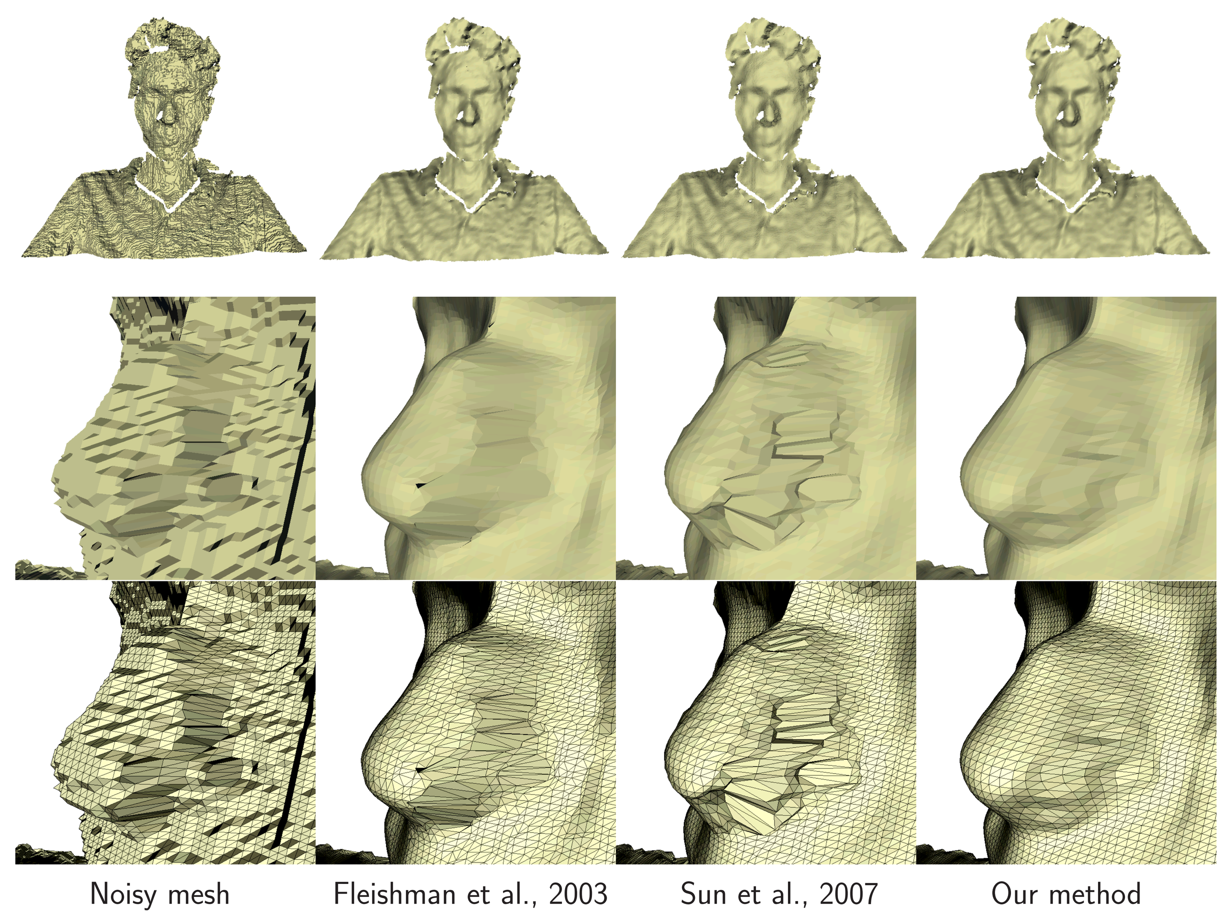
Denoised mesh quality of different methods on the bunny face corrupted with isotropic Gaussian noise ($\sigma = 0.2 \times$ mean edge length). The columns correspond to the ground truth, noisy mesh and solutions for Fleishman *et al.*, 2003, Jones *et al.*, 2003, Sun *et al.*, 2007, Zheng *et al.*, 2011 and our method respectively. First row shows surface quality. Second row shows folded face artefacts as black spots.

## Result: Synthetic Data (Contd.)

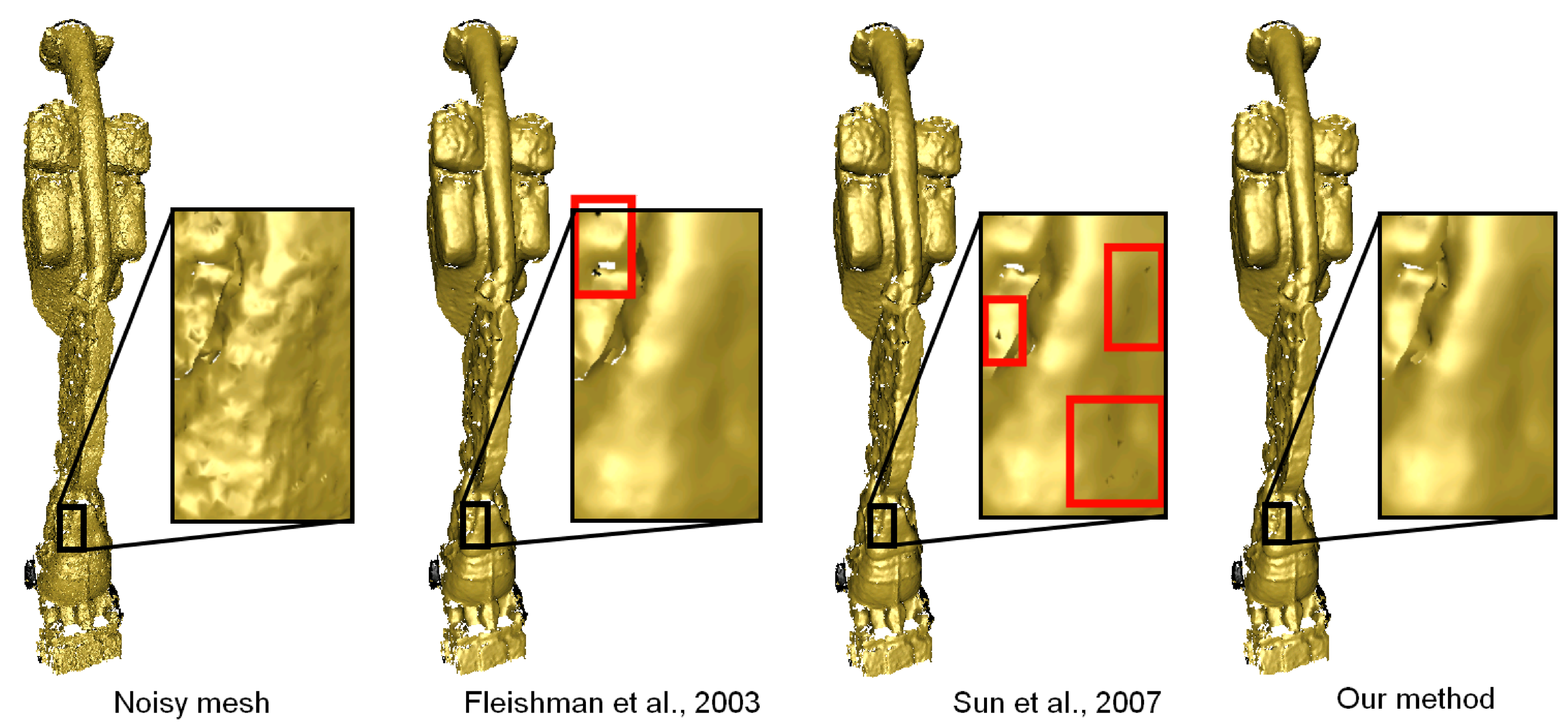
| Object | Error metric | Noisy | Bilateral | Jones | Sun | Zheng | Ours |
|---|---|---|---|---|---|---|---|
| Cube ($N_V = 1538, N_F = 3072$) $\sigma = 0.15 \times$ MEL | Mean NE (°) | 17.8359 | 15.8377 | 8.0962 | 0.6427 | 1.0038 | **0.4633** |
| | Mean VPE | 0.0339 | 0.0595 | 0.0359 | 0.0259 | NA | **0.0129** |
| Sphere ($N_V = 962, N_F = 1920$) $\sigma = 0.20 \times$ MEL | Mean NE (°) | 25.4718 | 9.5720 | 6.7752 | 5.6170 | 4.5514 | **3.0540** |
| | Mean VPE | 0.0404 | 0.0785 | 0.0383 | 0.0323 | NA | **0.0169** |
| Fandisk ($N_V = 6475, N_F = 12946$) $\sigma = 0.15 \times$ MEL | Mean NE (°) | 17.4080 | 14.9835 | 16.9082 | 2.8616 | **2.1682** | 4.7996 |
| | Mean VPE | 0.0098 | 0.0162 | 0.0382 | 0.0090 | NA | **0.0086** |
| Bunny face ($N_V = 15861, N_F = 31001$) $\sigma = 0.20 \times$ MEL | MeanNE (°) | 29.8166 | 13.6589 | 14.0275 | 13.6516 | 13.3182 | **8.0335** |
| | Mean VPE | 0.0340 | 0.0300 | 0.0304 | 0.0287 | NA | **0.0252** |
| Armadillo ($N_V = 67598, N_F = 134576$) $\sigma = 0.20 \times$ MEL | Mean NE (°) | 24.4805 | 9.0491 | **8.0203** | 8.2836 | 8.2653 | 9.0906 |
| | MeanVPE | 0.1669 | 0.1541 | 0.1435 | 0.1408 | NA | **0.1235** |

Comparison of denoising performance of our approach with other methods in the literature in terms of normal angle error (NE) and vertex position Euclidean error (VPE). 'NA' denotes 'Not Available' and 'MEL' denotes 'Mean Edge Length'.

## Result: Real Data




Denoising quality of different methods on a mesh generated from a raw Kinect depth map of a person. The first row shows the surface quality. The second and third rows show zoomed-in views on the nose region.




Denoising quality of different methods on a mesh of a sculptural pillar generated using multiview stereo applied to a set of images.

## Conclusion

- A **two-step denoising method** that globally solves for both normal mollification and vertex correction.
- Our vertex correction step accounts for **noise in all directions** and also incorporates a **novel cost function for enforcing face fairness** in the mesh.
- The ability of our method to provide **good mesh denoising while preserving face fairness** is demonstrated on a number of datasets.
- The **superiority of our approach** over other relevant methods in the literature is also established.

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